Magnetic components design

Note Title 10/24/2009

Magnetics

Review

$B \longrightarrow$ Flux density (Wb/m^2)

$H \longrightarrow$ Field intensity (A/m)

$\lambda \longrightarrow$ Flux linkage ($Wb \simeq V\,sec$)

$\phi \longrightarrow$ Flux (Wb) $\lambda = N\phi$

$M \longrightarrow$ Magnetization (A/m)

$J \longrightarrow$ current density (A/m^2)

$\nabla \times \vec{H} = \vec{J}_c$ (conduction currents) $\longrightarrow$ electric charges

$\nabla \times \vec{M} = \vec{J}_m$ (magnetizing currents) $\longrightarrow$ dipoles orientation when a magnetic field is applied

rotor $\Rightarrow$ indicate sources of vector fields

$\vec{J} = \vec{J}_c + \vec{J}_m$

$$\nabla \times \vec{B} = \mu_0 \vec{J} = \mu_0 (\vec{J}_c + \vec{J}_m) = \mu_0 \nabla \times (\vec{H} + \vec{M})$$

$$\vec{B} = \mu_0 (\vec{H} + \vec{M})$$

If a value χ_m is defined that relates $\vec{H}$ and $\vec{M}$ as $\Rightarrow$

$$\vec{M} = \chi_m \vec{H}$$

$\hookrightarrow$ magnetic susceptibility

Then

$$\vec{B} = \mu_0 \underbrace{(1+\chi_m)}_{\mu_r} \vec{H}$$

$$\vec{B} = \underbrace{\mu_0 \mu_r}_{\mu} \vec{H}$$

$$\vec{B} = \mu \vec{H}$$

μ ↳ magnetic permeability

Since $\chi_m = \chi_m(\vec{H})$ then $\mu = \mu(\vec{H})$

Materials:

Paramagnetics $\longrightarrow \mu > \mu_0$ or $\chi_m > 0$

Diamagnetics $\longrightarrow \mu < \mu_0$ or $\chi_m < 0$

Ferromagnetics $\longrightarrow \mu \gg \mu_0$ or $\chi_m \gg 0$

Vacuum $\longrightarrow \mu = \mu_0 = 4\pi \cdot 10^{-7}\ H/m$, $\chi_m = 0$

Curie temperature T_c $\rightarrow$ In ferromagnetics, for $T > T_c$, $\mu > \mu_0$ → they behave like paramagnetics

- General laws

From $\nabla \times \vec{H} = \vec{J}_c \longrightarrow \mathcal{F}_m = \oint_c \vec{H} d\vec{r} = Ni \rightarrow$ Ampere's law

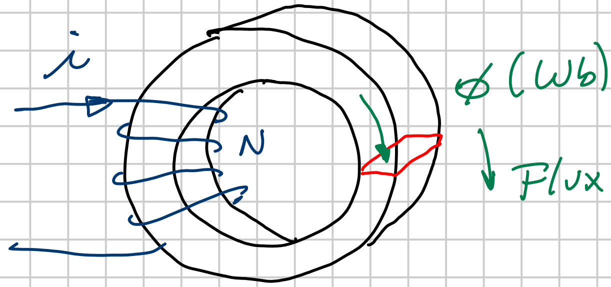
Ni ← $\int \vec{J} d\vec{s}$

$\mathcal{F}_m$ ↳ magnetomotive force

$\nabla \cdot \vec{B} = 0 \longrightarrow \oiint \vec{B} d\vec{s} = 0 \longrightarrow$ Gauss' law

$\nabla \times \vec{E} = -\dfrac{\partial \vec{B}}{\partial t} \longrightarrow \oint_c \vec{E} d\vec{r} = -\dfrac{d\phi}{dt} = \mathcal{E}$ (emf)

↳ Faraday's law



$$\phi\,(Wb) = BA$$

$B \to T \to$ Flux density

$$\oint_l \vec{H}\,d\vec{l} = \iint \vec{J}\,d\vec{s} \longrightarrow \mathcal{R}\,\phi = \mathcal{F}$$

$\phi \to$ Flux

$\mathcal{R} \to$ Reluctance $\left[\frac{1}{H}\right]$

$\mathcal{F} \to$ Magnetomotive force (MMF) $\to Ni$

Leackage flux

$\lambda \triangleq N\phi$

$N \to$ turns

$$\mathcal{R} = \frac{Ni}{\phi} = \frac{Ni}{\frac{\lambda}{N}} = \frac{N^2}{L}$$

with μ = constant

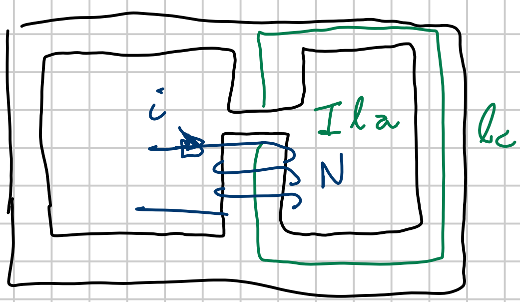
$$\mathcal{R} = \frac{Ni}{BA} = \frac{Ni}{\mu H A} = \frac{Ni}{\mu A \frac{Ni}{l}}$$

$$\boxed{\mathcal{R} = \frac{l}{\mu A}}$$

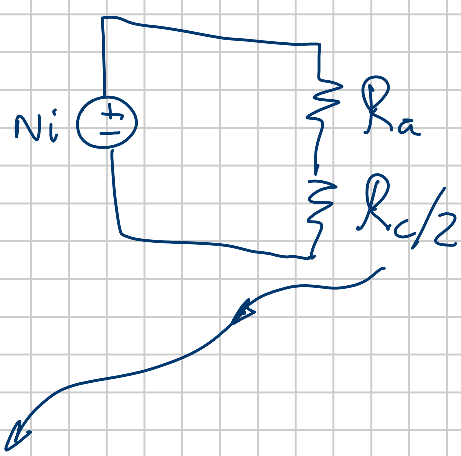
$$v = \frac{d\lambda}{dt} = \frac{d\lambda}{di}\frac{di}{dt} = L\frac{di}{dt}$$

$$L \triangleq \frac{d\lambda}{di} \xrightarrow{\text{if } \mu = \text{constant}} L = \frac{\lambda}{i}$$

Consider:



$\equiv$



This is an approximation in which I consider R_c each of the sides. The $\frac{1}{2}$ is because of the // magnetic circuit.

without airgap $R_a = 0$ → then the flux is determined by R_c

All of the energy is stored in the core

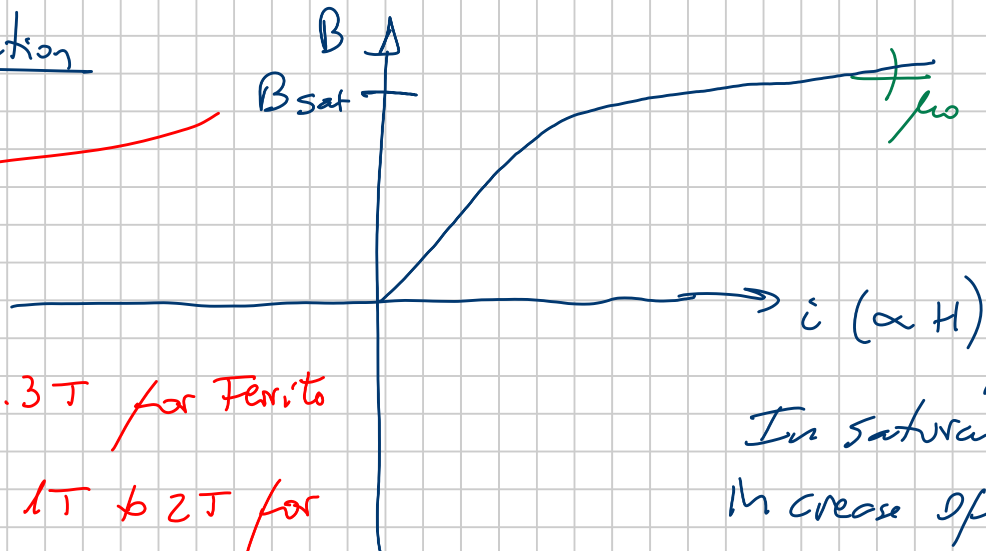
$\frac{1}{2} L i^2$

with airgap → $R_a \neq 0$ and typically $R_a >> R_c$
(because $\mu_c >> \mu_0 \to \mu_0$

then $\phi \simeq \frac{\mathcal{F}}{R_a}$

Most of the energy is stored in the air gap.

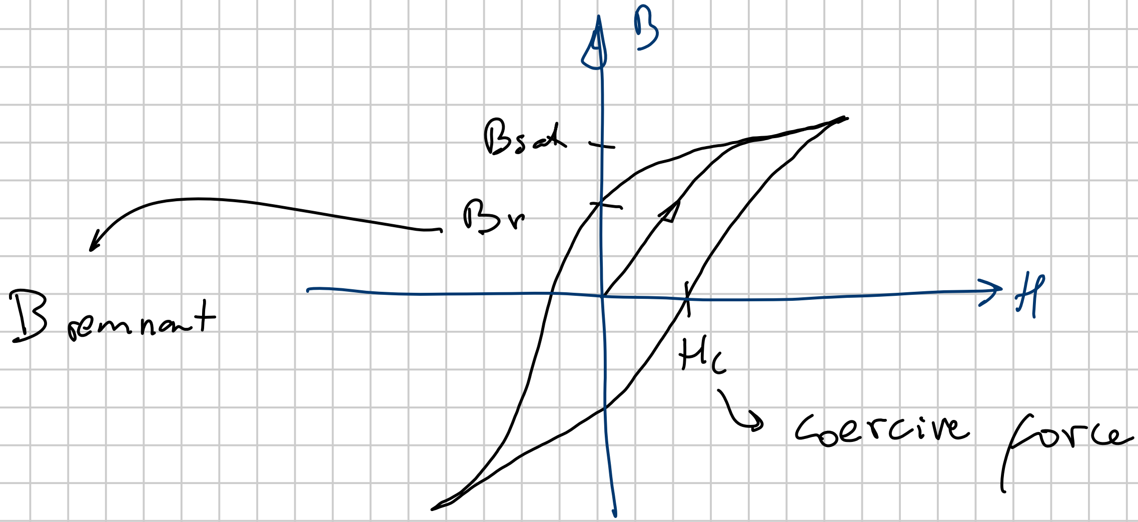
Saturation



$B_{sat} \simeq 0.3T$ for Ferrite

$B_{sat} \simeq 1T$ to $2T$ for powdered iron

In saturation little increase of "B" but significant increase in "i"



Energy density $\longrightarrow$ $w = \frac{1}{2}\frac{1}{\mu_c} B^2 \quad (J/m^3)$

The maximum energy I can store is limited by B_{sat}

without airgap $\longrightarrow$ $w_{w/og} = \frac{1}{2}\frac{B_{sat}^2}{\mu_c}$

with airgap $\longrightarrow$ $w_{wg} = \frac{1}{2}\frac{B_{sat}^2}{\mu_0}$

Since $\mu_c \gg \mu_0 \Longrightarrow w_{wg} > w_{w/og}$

- Saturation leads to several design limits

i) The MMF should not make B to exceed B_{sat}

$\mathcal{F} < \phi_{sat} \mathcal{R}$ It also implies an N_{max}

$Ni < B_{sat} A \mathcal{R} \longrightarrow \boxed{Ni_{max} = \frac{B_{sat}}{\mu_c} l}$

"Ampere-turn" limit

Usually manufacturers indicate $A_L = \frac{1}{\mathcal{R}} = \frac{L_{max}}{N^2}$

$$N i_{max} = B_{sat} A \mathcal{R} = \frac{B_{sat} A}{A_L}$$

So when designing a core one approach is to look for an A_L that makes $B < B_{sat}$ for a given N and i.

$$i_{max} = \frac{B_{sat} A}{N A_L}$$

2) $v = \dfrac{d\lambda}{dt}$

$$\lambda = N\phi = NBA = \underbrace{\int v\, dt}_{\text{volt-second rating}}$$

If $v = V_0 \cos \omega t \longrightarrow NBA = \dfrac{1}{\omega} V_0 \sin(\omega t)$

$$B_{max} = \frac{V_0}{\omega N A} < B_{sat} \longrightarrow \text{Max volt per turn limit}$$

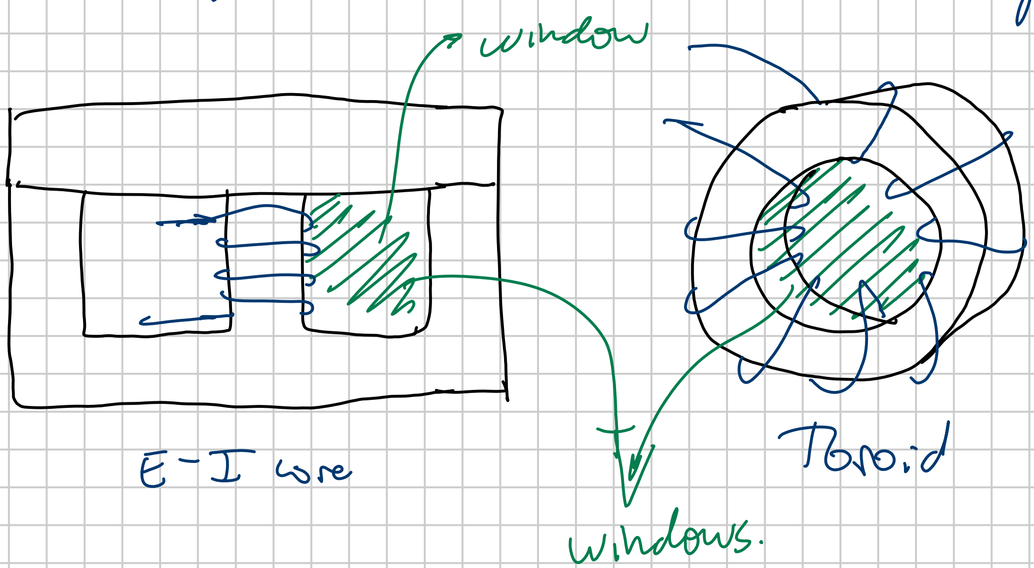
$$\frac{\int v\, dt}{NA} < B_{sat} \begin{cases} dc \longrightarrow V_{dc} \Delta t < B_{sat} N A_{core} \quad (\text{max volt-sec}) \\ ac \longrightarrow \dfrac{V_{peak}}{N} < B_{sat}\, \omega\, A_{core} \quad (\text{max volt/turn}) \end{cases}$$

The max volt-sec limit is what we need to consider in order to avoid saturating a boost converter inductor when the switch is closed.

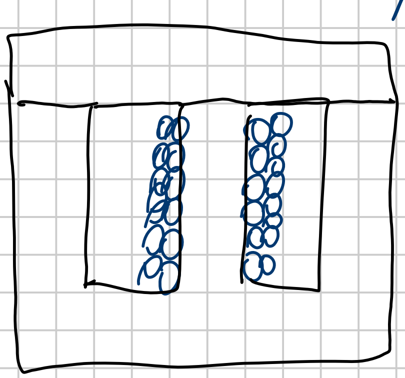
- Other design limits

From $N i_{max} = \frac{B_{sat}}{\mu} l$ I can obtain the diameter of the wire I can use (based on a max current density of about 300 A/cm^2)

This wire is then wound around a core with a given window



But the wire will occupy some space in the window

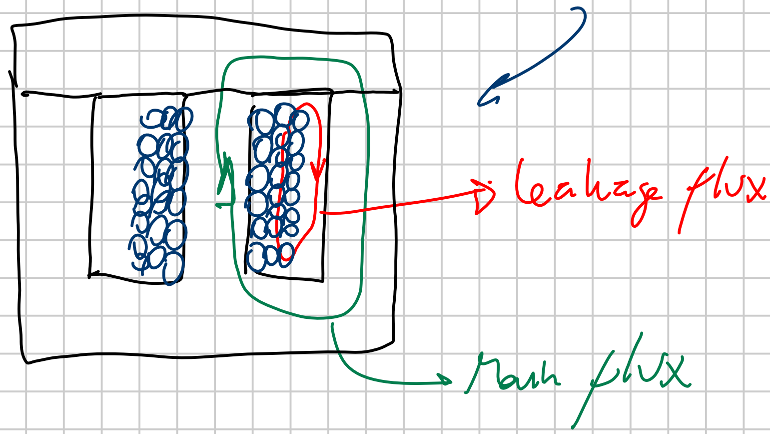


So we can define a fill factor

$$ff(\%) = \frac{N A_{wire}}{A_{window}} 100$$

Typically $ff = \begin{cases} 50\% \text{ for E-I, I, pot cores} \\ 10\% \text{ for toroids} \end{cases}$

higher ff may increase leakage inductance



creates undesirable voltage drops but sometimes it is useful b/c it helps to limit the current

Since $J = \dfrac{I}{A_{cond}}$

$\cancel{N}I = \cancel{N}J A_{wire}$

↳ Since there is a J_{max} (that avoids overheating the wire) then there is a NI_{max}, too

Another amp-turn limit.

Since $A_{wire} = \dfrac{ff(\%)}{100\,N} A_{window}$

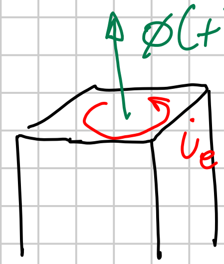
then $\boxed{J \leq \dfrac{N\,i_{max}\,100}{ff(\%)\,A_{wind}}}$

Typically $J \simeq 300\ A/cm^2$

Magnetic losses

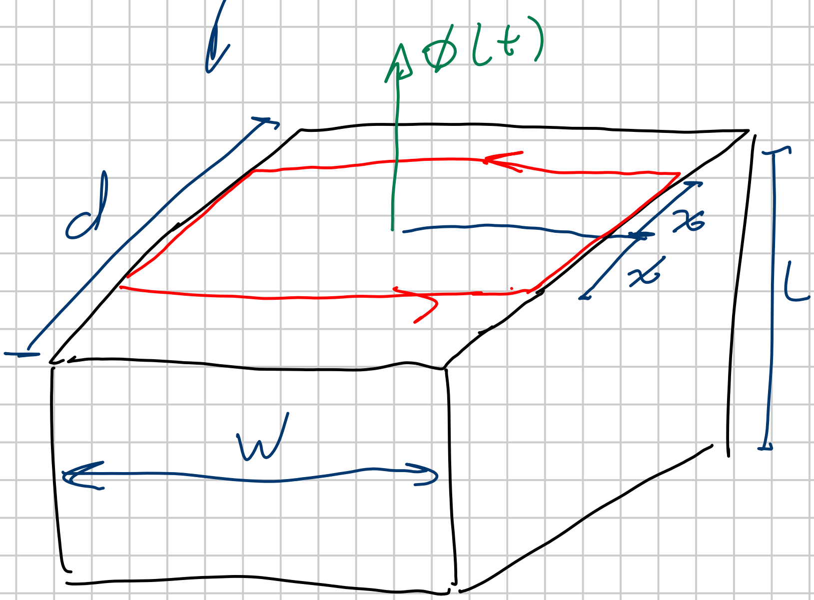
2 components → Eddy currents → currents that appear in the core because the magnetic material has no ∞ resistivity ρ

↳ hysteresis



$\rho_{ferrite}$ is high but $\rho_{powdered\ iron}$ is small

ρ_{iron} even smaller



$B(t) = B \sin(\omega t)$

in the red loop:

$$\phi(t) = \underbrace{2xW}_{A} B(t)$$

"N=1"

$$v(t) = \frac{d\phi}{dt} = 2xW\omega \cos\omega t$$

The resistance for a thin loop of thickness dx is

$$\left(R = \rho \frac{l}{S}\right) \longrightarrow r = \rho \frac{2(W+dx)}{L\,dx} \simeq \rho \frac{2W}{L\,dx}$$

Hence $p(t)$ (instantaneous power) is

$$p_{dx}(t) = \frac{v^2(t)}{r} = \frac{\left(2x\,W\,\omega \cos(\omega t)\right)^2}{\rho \frac{2W}{L\,dx}}$$

When integrated for dx between 0 and $d/2$ in order to obtain $p(t)$ for the entire core volume and then averaged to obtain P it yields

$$P_e = \frac{W L d^3 \omega^2 B^2}{24\rho}$$

or, per unit volume it is

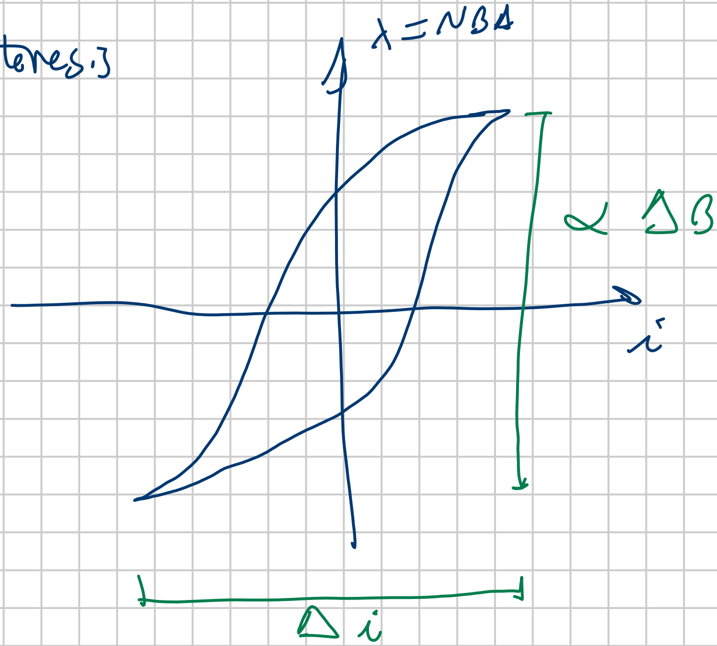
$$P_{e,sp} = \frac{d^2 \omega^2 B^2}{24\rho}$$

higher frequencies yield higher losses

laminations help reduce eddy current losses

- steel with lamination not useful for $f >$ few kHz
- Ferrites good for 10kHz — 10MHz
- Powdered iron $\longrightarrow$ good for high frequencies but not so high as ferrites
  - $\longrightarrow$ high B_{sat}

→ Hysteresis



Every time we try to invert the magnetizing direction of the dipoles that make a magnetic material we need to provide energy to the core. This energy is used to alter the magnetic cristaline structure in an irreversible way and will never be recover

Notice that

$$\underbrace{\int_{i(0)}^{i(t)} \lambda \, di}_{\text{Area in the } \lambda = \lambda(i) \text{ curve}} \longrightarrow \int_{i(0)}^{i(t)} \frac{d\lambda}{dt} dt \, di = \underbrace{\underbrace{\int_{i(0)}^{i(t)} v \, di . dt}_{\text{power}}}_{\text{energy}}$$

So the area of the hysteresis loop is proportional to the energy I loose in the irreversible process of reorient the dipoles.

An empirical form for hysteresis losses is:

$$P_{h,sp} = k f^{a} (B_{ac})^{d}$$

d: usually between 2 and 3

$B_{ac} = \frac{\Delta B}{2}$

a: usually between 1 and 2

k: in the order of 10^{-6}

$P_{h,sp} = \left[\frac{mW}{cm^3}\right]$

$f = [kHz]$

$B_{ac} = [mT]$

A general form for losses:

www.micrometals.com

$$P_{core} = \frac{V_{core}}{1000}\left[\frac{f}{\frac{a}{B_{ac}^{3}} + \frac{b}{B_{ac}^{2.3}} + \frac{c}{B_{ac}^{1.65}}} + d f^{2} B_{ac}^{2}\right]$$

hysteresis

Eddie

a, b, c, d: depends on material.

V_{core} → volume of core

B_{ac} → peak value for flux density in Gauss

$$V_L = E = \frac{\Delta\lambda}{\Delta t} = \frac{N A \Delta B}{\Delta t} = \frac{2 N A B_{ac}}{D T} = \frac{2 N A B_{ac} f}{D}$$

$$B_{ac} f = \frac{E D}{2 N A}$$

$B_{ac} \rightarrow 0 - 200G$

$B_{sat} = 1.38T$

$L = N^2 A_L$

$$\left(\frac{\Delta i}{\Delta \lambda} = \frac{1}{L}\right)$$

$1T = 10000\,G$ ← CGS

$$\Delta i = \frac{2 A N B_{ac}}{L}$$

Consider a boost converter inductor

$$\Delta i = \frac{E}{L}\frac{D}{f}$$

But $L i = N B A \longrightarrow L(\Delta i) = N(\Delta B)A$

$$\Delta i = \left(\frac{NA}{L}\right)\Delta B$$

So $\Delta B = \frac{E D}{f} \frac{1}{NA}$

Also $\rightarrow P_{h,sp} = k f^{a} (B_{ac})^{d}$

$$B_{ac} = \frac{\Delta B}{2}$$

$$P_{h,sp} = k f^{a} \left(\frac{\Delta B}{2}\right)^{d}$$

$$P_{h,sp} = k f^{a} \left(\frac{E D}{f}\frac{1}{NA}\right)^{d} = k f^{a-d}\left(\frac{ED}{NA}\right)^{d}$$

$P_h \propto \frac{1}{N^{d}}$ $\rightarrow$ let's say 2

Winding ohmic losses $\longrightarrow R = \rho_{cu} \frac{l_{wire}}{A_{wire}}$

$$l_{wire} = N\, l_{turn}$$

$P_{ohmic} \propto N$