EE411 – Summer 2008, Final Exam

Please, show all your work on the test sheets. A correct answer without supporting work gets no credit. One sheet of notes is permitted. You can also use your sheet of notes from tests #1 and #2. Write your name in all pages. Do not un-staple. You have 90 minutes to complete the test.

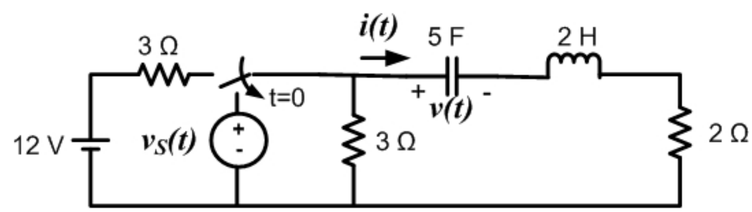
Problem 1 (30 points)

Consider the second order circuit in the figure and that the dc voltage source has been connected for a long time. Please answer the following questions:

a) Write down the complete form of *v(t)* and *i(t)* from the time the switch is closed.

b) Calculate the phasors **V** corresponding to *v(t)* and the phasor **I** corresponding to *i(t)* a long time after the switch was closed.

c) Compare the answers for the two previous questions. What conclusions can you draw from the comparison?



$v_S(t)=10\cos(2\cancel{\pi}t)$

$i(0)=0$

$V(0) = V_{R_{3\Omega}} = 6\,V$

When the switch is closed

$$L\frac{di}{dt} + Ri + \underbrace{\frac{1}{C}\int i\,dt + V(0)}_{v_c(t)} = 10\cos(2t)$$

$$\frac{d^2i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{1}{LC}i = \frac{-20}{2}\sin(2t)$$

$$s^2 + \frac{2}{2}s + \frac{1}{10} = 0$$

$$s = \frac{-1 \pm \sqrt{1-0.4}}{2} = \frac{-1 \pm 0.775}{2} = \begin{cases} -0.887 \\ -0.113 \end{cases}$$

$$i_h(t) = A_1 e^{-0.887t} + A_2 e^{-0.113t}$$

$$i_f(t) = A_3 \sin 2t + A_4 \cos 2t, \quad \frac{d^2 i}{dt^2} + \underbrace{\frac{R}{L}}_{1}\frac{di}{dt} + \underbrace{\frac{1}{LC}}_{0.1} i = -20\sin(2t)$$

$$-4A_3 \sin 2t - 4A_4 \cos 2t + 2A_3 \cos 2t - 2A_4 \sin 2t + 0.1 A_3 \sin 2t +$$

$$+ 0.1 A_4 \cos 2t = -20 \sin 2t$$

$$\begin{cases} -4A_3 - 2A_4 + 0.1A_3 = -20 \longrightarrow -3.9A_3 - 2A_4 = -\frac{20}{2} \\ -4A_4 + 2A_3 + 0.1A_4 = 0 \longrightarrow -3.9A_4 + 2A_3 = 0 \end{cases}$$

$$A_3 = \frac{3.9}{2} A_4 = 1.95 A_4$$

$$-3.9 \times 1.95 A_4 - 2A_4 = -10$$

$$A_4 = \frac{20}{9.605} = 1.041$$

$$A_3 = 2.03$$

$$\boxed{i_f(t) = 2.03 \sin 2t + 1.041 \cos 2t = 2.28 \cos(2t - 62.85)} \quad (1)$$

$$i(t) = A_1 e^{-0.887t} + A_2 e^{-0.113t} + 2.28 \cos(2t - 62.85)$$

$$i(0) = 0 = A_1 + A_2 + 2.28 \cos(-62.85) = A_1 + A_2 + 1.041$$

$$v(0) = 6 = 10 - v_L(0) - v_R(0) = 10 - L\frac{di}{dt}\Big|_{t=0} - Ri\Big|_{t=0} =$$

$$= 10 - 2(-0.887A_1 - 0.113A_2 + 4.055) - 2(A_1 + A_2 + 1.041) =$$

$$= -(2 \times 2.28 \times \sin(-62.85))$$

see next page

$$6 = -0.226A_1 - 1.774A_2 - 0.192$$

$$\begin{cases} -0.226A_1 - 1.774A_2 = 6.192 \\ A_1 + A_2 = -1.041 \end{cases}$$

$$A_1 = 2.807$$

$$A_2 = -3.85$$

$$i(t) = 2.807e^{-0.887t} - 3.85\ e^{-0.113t} + 2.28\cos(2t - 62.85)$$

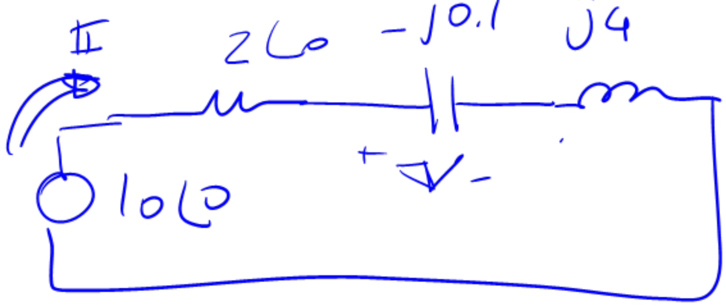
$$v(t) = \frac{1}{C}\int_0^t i\,dt - 6 =$$

$$= \frac{1}{5}\left(\frac{2.807}{-0.887}e^{-0.887t} + \frac{3.85}{0.113}e^{-0.113t} + \frac{2.28}{2}\sin(2t - 62.85)\right)\Big|_0^t - 6 =$$

$$= \frac{1}{5}\left(-3.165\ e^{-0.887t} + \cancel{\text{[illegible]}}\ e^{-0.113t} + 1.14\sin(2t - 62.85)\right)\Big|_0^t - 6 =$$

$$\boxed{v(t) = -0.633e^{-0.887} + 6.814e^{-0.113t} + \underbrace{0.228\sin(2t - 62.85)}_{(2)}}$$

b)



$$I = \frac{10\angle 0}{2\angle 0 + 0.1\angle -90 + 4\angle 90} = \frac{10\angle 0}{4.38\angle 62.85} = 2.28\angle -62.85$$

Same as in (1)

$$V = I\ 0.1\angle -90 = \underbrace{0.228\angle -152.85}_{\text{Same as in (2)}}$$

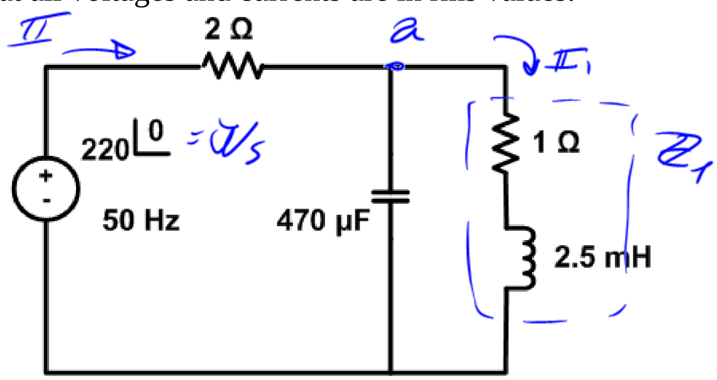
EE411 – Summer 2008, Final Exam

Problem 2 (20 points)

For the circuit in the figure calculate.

a) Active or reactive power of each of the passive elements as appropriate.
b) Verify the conservation of both active and reactive power by calculating the complex power generated by the source.

You can assume that all voltages and currents are in rms values.



$\omega = 2\pi f = 2 \cdot \pi \, 50 = 314.16$

$X_C = -\dfrac{1}{\omega C} = -\dfrac{1}{314.16 \; 470 \cdot 10^{-6}} = -6.77$

$X_L = \omega L = 314.16 \cdot 2.5 \cdot 10^{-3} = 0.7855$

$Z_1 = 1 + j\,0.7854 = 1.27\angle 38.14$

$Z_{eq} = (jX_C) \,//\, Z_1 = \dfrac{1}{\dfrac{1}{1.27\angle 38.14} + \dfrac{1}{6.77\angle -90}} = 1.418\angle 28.65$

$Z_T = 2\angle 0 + 1.418\angle 28.65 = 3.31\angle 11.84$

$I = \dfrac{220\angle 0}{3.31\angle 11.84} = 66.35\angle -11.84$

$V_a = V_s - I \, 2\angle 0 = 94.14\angle 16.81$

$I_1 = \dfrac{V_a}{Z_1} = \dfrac{94.14\angle 16.81}{1.27\angle 38.14} = 74.12\angle -21.32$

$$P_{R2\Omega} = 2I^2 = 2(66.35)^2 = 8804.64 \text{ W}$$

$$Q_C = \frac{V_a^2}{X_C} = \frac{(94.14)^2}{-6.77} = -1309 \text{ VAr}$$

$$P_{R1\Omega} = I_1^2(1) = (74.12)^2 \, 1 = 5493.77 \text{ W}$$

$$Q_L = I_1^2 X_L = (74.12)^2 \cdot 0.7854 = 4314.8 \text{ W}$$

$$S_s = V_s I^* = 220\angle 0 \; 66.35\angle +11.84 = 14597\angle 11.84 =$$

$$= 14286.44 + j\,2995$$

$$P_{R2\Omega} + P_{R1\Omega} = 14298.41 \simeq$$

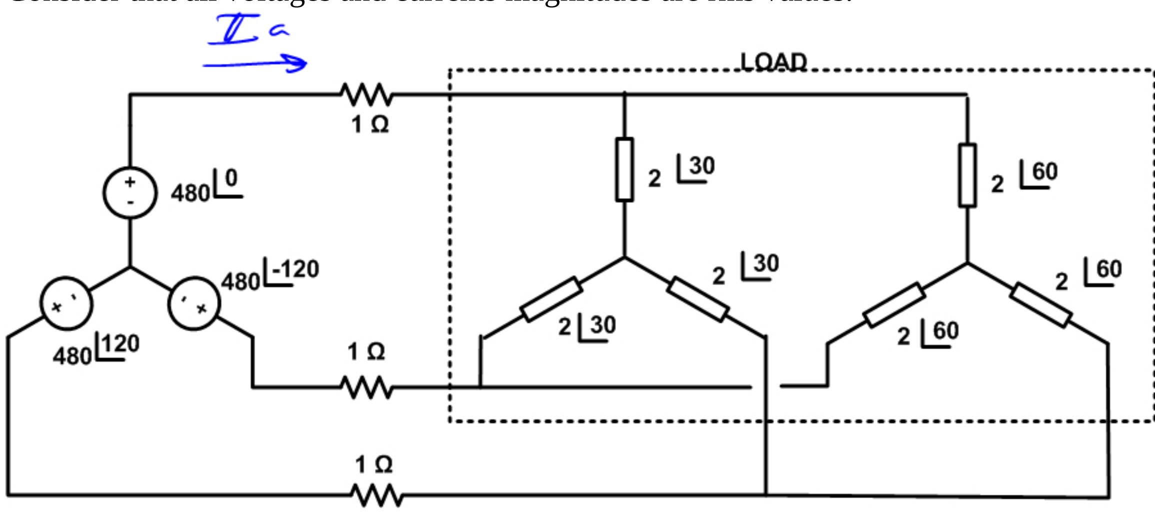
$$Q_L + Q_C = 4314.8 - 1309 = 3005.8 \simeq$$

EE411 – Summer 2008, Final Exam

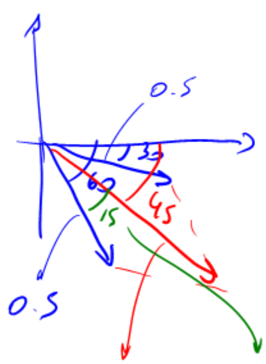
Problem 3 (25 points)
For the 3-phase circuit in the figure calculate:

a) Load's phase and line voltage phasors (all 6 of them).
b) Load's phase and line current phasors (all 6 of them)..
c) Load's complex power.

Consider that all voltages and currents magnitudes are rms values.



$$Z_{Leq} = 2\angle 30 \parallel 2\angle 60 = \frac{1}{\frac{1}{2\angle 60} + \frac{1}{2\angle 30}} = \frac{1}{0.5\angle -60 + 0.5\angle -30} =$$



$$= \frac{1}{0.966\angle -45} = 1.03\angle 45$$

2 0.5 cos 15 = 0.966

$$Z_T = Z_\ell + Z_{Leq} = 1\angle 0 + 1.03\angle 45 = 1.88\angle 22.9$$

$$I_a = \frac{480\angle 0}{1.88\angle 22.9} = 255.26\angle -22.9$$

$$I_b = 255.26\angle -142.9$$

$$I_c = 255.26\angle -262.9$$

Phase ~~and~~ and line ~~~~ currents are the same

$$V_{an} = V_{20} - I Z_e = 480\angle 0 - 1\angle 0 \; 255.26\angle -22.9 =$$

$$= 264.27\angle 22.1$$

$$V_{bn} = 264.27\angle -97.9$$

$$V_{cn} = 264.27\angle -217.9$$

$$V_{ab} = V_{an}\sqrt{3}\angle 30 = 457.26\angle 52.1$$

$$V_{bc} = V_{bn}\sqrt{3}\angle 30 = 457.26\angle -67.9$$

$$V_{ca} = V_{cn}\sqrt{3}\; 30 = 457.26\angle -187.9$$

$$P = \sqrt{3}\, V_L I_L \cos\varphi_{eq} = \sqrt{3}\; 457.26\; 255.26 \cos 45 =$$

$$= 142952.9 \, W$$

$$Q = \sqrt{3}\, V_L I_L \sin\varphi_{eq} = 142952.9 \; VAr.$$

$$S = 142952.9\,(1+j) = 202165.9\angle 45 \; VA$$

EE411 – Summer 2008, Final Exam

Problem 4 (25 points)
Sketch the Bode plot for the magnitude of the following transfer function

$$\mathbf{H}(s) = \frac{5s(100 - s^2)}{(s+100)(s^2+20{,}000s+10^8)}$$

$$H(s) = \frac{-5s(10+s)(s-10)}{(s+100)(s+10^4)^2} = -\frac{5\cdot\cancel{10\cdot 10}\left(\frac{s}{10}+1\right)\left(\frac{s}{10}-1\right)}{\cancel{100}\left(\frac{s}{100}+1\right)10^8\left(\frac{s}{10^4}+1\right)^2}$$