Critical values of the F distribution for a 99% confidence level

	n_1						
n_2	1	2	3	4	5	6	10
1	4,052.181	4,999.500	5,403.352	5,624.583	5,763.650	5,858.986	6,055.847
2	98.503	99.000	99.166	99.249	99.299	99.333	99.399
3	34.116	30.817	29.457	28.710	28.237	27.911	27.229
4	21.198	18.000	16.694	15.977	15.522	15.207	14.546
5	16.258	13.274	12.060	11.392	10.967	10.672	10.051
6	13.745	10.925	9.780	9.148	8.746	8.466	7.874
10	10.044	7.559	6.552	5.994	5.636	5.386	4.849
20	8.096	5.849	4.938	4.431	4.103	3.871	3.368
40	7.314	5.179	4.313	3.828	3.514	3.291	2.801
60	7.077	4.977	4.126	3.649	3.339	3.119	2.632
80	6.963	4.881	4.036	3.563	3.255	3.036	2.551
100	6.895	4.824	3.984	3.513	3.206	2.988	2.503
200	6.763	4.713	3.881	3.414	3.110	2.893	2.411
500	6.686	4.648	3.821	3.357	3.054	2.838	2.356

	n_1						
n_2	20	40	60	80	100	200	500
1	6,208.730	6,286.782	6,313.030	6,326.197	6,334.110	6,349.967	6,359.501
2	99.449	99.474	99.482	99.487	99.489	99.494	99.497
3	26.690	26.411	26.316	26.269	26.240	26.183	26.148
4	14.020	13.745	13.652	13.605	13.577	13.520	13.486
5	9.553	9.291	9.202	9.157	9.130	9.075	9.042
6	7.396	7.143	7.057	7.013	6.987	6.934	6.902
10	4.405	4.165	4.082	4.039	4.014	3.962	3.930
20	2.938	2.695	2.608	2.563	2.535	2.479	2.445
40	2.369	2.114	2.019	1.969	1.938	1.874	1.833
60	2.198	1.936	1.836	1.783	1.749	1.678	1.633
80	2.115	1.849	1.746	1.690	1.655	1.579	1.530
100	2.067	1.797	1.692	1.634	1.598	1.518	1.466
200	1.971	1.694	1.583	1.521	1.481	1.391	1.328
500	1.915	1.633	1.517	1.452	1.408	1.308	1.232

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Appendix C Selected statistical tables

'There are three types of mathematicians. Those who can count, and those who cannot.'

Robert Arthur

C.1 Critical values of Student's t distribution

The derivation of the t distribution was first published by W. S. Gosset in 1908. Gosset was employed by an Irish brewery that did not allow its employees to publish the results of their research. Gosset instead published his work using the pseudonym 'Student.' Consequently, the t distribution is often referred to as *Student's t distribution*.

The t distribution is commonly used for calculating confidence intervals. It is bell-shaped and symmetric around a mean of zero, similar to the Gaussian (normal) distribution. The t distribution has a higher variance than a Gaussian distribution, making it appear flatter and more spread out. As the number of degrees of freedom in the t distribution becomes very large, it is well approximated by a Gaussian distribution with a mean of 0 and a variance of 1, that is, the so-called 'standard normal distribution.' Consequently, the last line in the following table shows the critical values for a standard normal distribution.

The values shown in the following table are the critical values of the t distribution. Each $t_{a;n}$ value in the table is the value on the x axis for which there is an area of a to the left of $t_{a;n}$, as shown in Figure C.1, where n is the number of degrees of freedom. For example, say you want to find the t value necessary to compute a 95% confidence interval with eight degrees of freedom. The corresponding significance level is $\alpha = 1 - 0.95 = 0.05$. Since the distribution is symmetric around zero, but the table shows the area to the left of $t_{a;n}$, we must use $1 - \alpha/2 = 1 - 0.05/2 = 0.975$ as the area a for finding the critical t value. The necessary value from the table is then $t_{0.975;8} = 2.306$.

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Critical values of the t distribution

n	a 0.7000	0.8000	0.9000	0.9500	0.9750	0.9950	0.9995
1	0.727	1.376	3.078	6.314	12.706	63.657	636.619
2	0.617	1.061	1.886	2.920	4.303	9.925	31.599
3	0.584	0.979	1.638	2.354	3.183	5.841	12.924
4	0.569	0.941	1.533	2.132	2.777	4.604	8.610
5	0.559	0.920	1.476	2.015	2.571	4.032	6.869
6	0.553	0.906	1.440	1.943	2.447	3.708	5.959
7	0.549	0.896	1.415	1.895	2.365	3.500	5.408
8	0.546	0.889	1.397	1.860	2.306	3.356	5.041
9	0.544	0.883	1.383	1.833	2.262	3.250	4.781
10	0.542	0.879	1.372	1.813	2.228	3.169	4.587
11	0.540	0.876	1.363	1.796	2.201	3.106	4.437
12	0.539	0.873	1.356	1.782	2.179	3.055	4.318
13	0.538	0.870	1.350	1.771	2.161	3.012	4.221
14	0.537	0.868	1.345	1.761	2.145	2.977	4.141
15	0.536	0.866	1.341	1.753	2.132	2.947	4.073
16	0.535	0.865	1.337	1.746	2.120	2.921	4.015
17	0.534	0.863	1.333	1.740	2.110	2.898	3.965
18	0.534	0.862	1.330	1.734	2.101	2.879	3.922
19	0.533	0.861	1.328	1.729	2.093	2.861	3.884
20	0.533	0.860	1.325	1.725	2.086	2.845	3.850
21	0.533	0.859	1.323	1.721	2.080	2.832	3.819
22	0.532	0.858	1.321	1.717	2.074	2.819	3.792
23	0.532	0.858	1.320	1.714	2.069	2.807	3.768
24	0.531	0.857	1.318	1.711	2.064	2.797	3.746
25	0.531	0.856	1.316	1.708	2.060	2.788	3.725
26	0.531	0.856	1.315	1.706	2.056	2.779	3.707
27	0.531	0.855	1.314	1.703	2.052	2.771	3.690
28	0.530	0.855	1.313	1.701	2.048	2.763	3.674
29	0.530	0.854	1.311	1.699	2.045	2.757	3.660
30	0.530	0.854	1.310	1.697	2.042	2.750	3.646
40	0.529	0.851	1.303	1.684	2.021	2.705	3.551
50	0.528	0.849	1.299	1.676	2.009	2.678	3.496
60	0.527	0.848	1.296	1.671	2.000	2.660	3.460
70	0.527	0.847	1.294	1.667	1.994	2.648	3.435
80	0.527	0.846	1.292	1.664	1.990	2.639	3.416
90	0.526	0.846	1.291	1.662	1.987	2.632	3.402
100	0.526	0.845	1.290	1.660	1.984	2.626	3.391
110	0.526	0.845	1.289	1.659	1.982	2.621	3.381
120	0.526	0.845	1.289	1.658	1.980	2.618	3.374
130	0.526	0.844	1.288	1.657	1.978	2.614	3.367
140	0.526	0.844	1.288	1.656	1.977	2.612	3.362
150	0.526	0.844	1.287	1.655	1.976	2.609	3.357

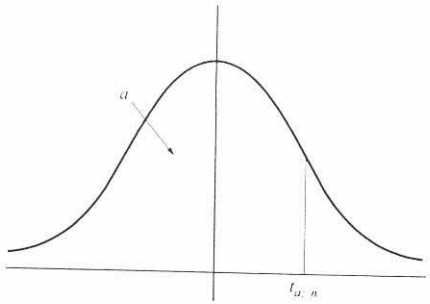

Figure C.1 Each $t_{a;n}$ value in the t table is the value on the x axis for which there is an area of a to the left of $t_{a;n}$.

C.2 Critical values of the F distribution

The F-test is used to determine whether two variances are significantly different. The F *statistic* is the ratio of the two variances, s_1^2 and s_2^2:

$$F_{\text{calculated}} = \frac{s_1^2}{s_2^2}. \tag{C.1}$$

where variance s_1^2 has n_1 degrees of freedom and variance s_2^2 has n_2 degrees of freedom. If $F_{\text{calculated}}$ is larger than the critical value obtained from one of the following tables, we can conclude, with the confidence level shown for the table used, that there is a statistically significant difference between the variances.

For example, given $s_1^2 = 23.12$ and $s_2^2 = 3.44$ with $n_1 = 4$ and $n_2 = 20$, we find $F_{\text{calculated}} = 23.12/3.44 = 6.72$. The critical value from the 90% confidence-level F distribution table is $F_{0.90;4,20} = 2.249$. Since $F_{\text{calculated}} > F_{0.90;4,20}$, we conclude with 90% confidence that there is a statistically significant difference between the variances.

Critical values of the F distribution for a 90% confidence level

	n_1						
n_2	1	2	3	4	5	6	10
1	39.863	49.500	53.593	55.833	57.240	58.204	60.195
2	8.526	9.000	9.162	9.243	9.293	9.326	9.392
3	5.538	5.462	5.391	5.343	5.309	5.285	5.230
4	4.545	4.325	4.191	4.107	4.051	4.010	3.920
5	4.060	3.780	3.619	3.520	3.453	3.405	3.297
6	3.776	3.463	3.289	3.181	3.108	3.055	2.937
10	3.285	2.924	2.728	2.605	2.522	2.461	2.323
20	2.975	2.589	2.380	2.249	2.158	2.091	1.937
40	2.835	2.440	2.226	2.091	1.997	1.927	1.763
60	2.791	2.393	2.177	2.041	1.946	1.875	1.707
80	2.769	2.370	2.154	2.016	1.921	1.849	1.680
100	2.756	2.356	2.139	2.002	1.906	1.834	1.663
200	2.731	2.329	2.111	1.973	1.876	1.804	1.631
500	2.716	2.313	2.095	1.956	1.859	1.786	1.612

	n_1						
n_2	20	40	60	80	100	200	500
1	61.740	62.529	62.794	62.927	63.007	63.167	63.264
2	9.441	9.466	9.475	9.479	9.481	9.486	9.489
3	5.184	5.160	5.151	5.147	5.144	5.139	5.136
4	3.844	3.804	3.790	3.782	3.778	3.769	3.764
5	3.207	3.157	3.140	3.132	3.126	3.116	3.109
6	2.836	2.781	2.762	2.752	2.746	2.734	2.727
10	2.201	2.132	2.107	2.095	2.087	2.071	2.062
20	1.794	1.708	1.677	1.660	1.650	1.629	1.616
40	1.605	1.506	1.467	1.447	1.434	1.406	1.389
60	1.543	1.437	1.395	1.372	1.358	1.326	1.306
80	1.513	1.403	1.358	1.334	1.318	1.284	1.261
100	1.494	1.382	1.336	1.310	1.293	1.257	1.232
200	1.458	1.339	1.289	1.261	1.242	1.199	1.168
500	1.435	1.313	1.260	1.229	1.209	1.160	1.122

Critical values of the F distribution for a 95% confidence level

	n_1						
n_2	1	2	3	4	5	6	10
1	161.448	199.500	215.707	224.583	230.162	233.986	241.882
2	18.513	19.000	19.164	19.247	19.296	19.330	19.396
3	10.128	9.552	9.277	9.117	9.013	8.941	8.786
4	7.709	6.944	6.591	6.388	6.256	6.163	5.964
5	6.608	5.786	5.409	5.192	5.050	4.950	4.735
6	5.987	5.143	4.757	4.534	4.387	4.284	4.060
10	4.965	4.103	3.708	3.478	3.326	3.217	2.978
20	4.351	3.493	3.098	2.866	2.711	2.599	2.348
40	4.085	3.232	2.839	2.606	2.449	2.336	2.077
60	4.001	3.150	2.758	2.525	2.368	2.254	1.993
80	3.960	3.111	2.719	2.486	2.329	2.214	1.951
100	3.936	3.087	2.696	2.463	2.305	2.191	1.927
200	3.888	3.041	2.650	2.417	2.259	2.144	1.878
500	3.860	3.014	2.623	2.390	2.232	2.117	1.850

	n_1						
n_2	20	40	60	80	100	200	500
1	248.013	251.143	252.196	252.724	253.041	253.677	254.059
2	19.446	19.471	19.479	19.483	19.486	19.491	19.494
3	8.660	8.594	8.572	8.561	8.554	8.540	8.532
4	5.803	5.717	5.688	5.673	5.664	5.646	5.635
5	4.558	4.464	4.431	4.415	4.405	4.385	4.373
6	3.874	3.774	3.740	3.722	3.712	3.690	3.678
10	2.774	2.661	2.621	2.601	2.588	2.563	2.548
20	2.124	1.994	1.946	1.922	1.907	1.875	1.856
40	1.839	1.693	1.637	1.608	1.589	1.551	1.526
60	1.748	1.594	1.534	1.502	1.481	1.438	1.409
80	1.703	1.545	1.482	1.448	1.426	1.379	1.347
100	1.676	1.515	1.450	1.415	1.392	1.342	1.308
200	1.623	1.455	1.386	1.346	1.321	1.263	1.221
500	1.592	1.419	1.345	1.303	1.275	1.210	1.159