Fuzzy Logic in Control Systems: Fuzzy Logic Controller—Part I

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Abstract—During the past several years, fuzzy control has emerged as one of the most active and fruitful areas for research in the applications of fuzzy set theory, especially in the realm of industrial processes, which do not lend themselves to control by conventional methods because of a lack of quantitative data regarding the input–output relations. Fuzzy control is based on fuzzy logic—a logical system which is much closer in spirit to human thinking and natural language than traditional logical systems. The fuzzy logic controller (FLC) based on fuzzy logic provides a means of converting a linguistic control strategy based on expert knowledge into an automatic control strategy. A survey of the FLC is presented; a general methodology for constructing an FLC and assessing its performance is described; and problems that need further research are pointed out. In particular, the exposition includes a discussion of fuzzification and defuzzification strategies, the derivation of the database and fuzzy control rules, the definition of fuzzy implication, and an analysis of fuzzy reasoning mechanisms.

I. Introduction

During the past several years, fuzzy control has emerged as one of the most active and fruitful areas for research in the application of fuzzy set theory [141]. The pioneering research of Mamdani and his colleagues on fuzzy control [63]–[66], [50] was motivated by Zadeh's seminal papers on the linguistic approach and system analysis based on the theory of fuzzy sets [142], [143], [145], [146]. Recent applications of fuzzy control in water quality control [127], [35], automatic train operation systems [135], [136], [139], automatic container crane operation systems [137]–[139], elevator control [23], nuclear reactor control [4], [51], automobile transmission control [40], fuzzy logic controller hardware systems [130], [131], fuzzy memory devices [107], [108], [120], [128], [129], [133], and fuzzy computers [132] have pointed a way for an effective utilization of fuzzy control in the context of complex ill-defined processes that can be controlled by a skilled human operator without the knowledge of their underlying dynamics.

The literature in fuzzy control has been growing rapidly in recent years, making it difficult to present a comprehensive survey of the wide variety of applications that have been made. Historically, the important milestones in the development of fuzzy control may be summarized as shown in table I. It should be stressed, however, the choice of the milestones is a subjective matter.

Fuzzy logic, which is the logic on which fuzzy control is based, is much closer in spirit to human thinking and natural language than the traditional logical systems. Basically, it provides an effective means of capturing the approximate, inexact nature of the real world. Viewed in this perspective, the essential part of the fuzzy logic controller (FLC) is a set of linguistic control rules related by the dual concepts of fuzzy implication and the compositional rule of inference. In essence, then, the FLC provides an algorithm which can convert the linguistic control strategy based on expert knowledge into an automatic control strategy. Experience shows that the FLC yields results superior to those obtained by conventional control algorithms. In particular, the methodology of the FLC appears very useful when the processes are too complex for analysis by conventional quantitative techniques or when the available sources of information are interpreted qualitatively, inexactly, or uncertainly. Thus fuzzy logic control may be viewed as a step toward a rapprochement between conventional precise mathematical control and human-like decision making, as indicated by Gupta [30].

However, at present there is no systematic procedure for the design of an FLC. In this paper we present a survey of the FLC methodology and point to the problems which need further research. Our investigation includes fuzzification and defuzzification strategies, the derivation of the database and fuzzy control rules, the definition of a fuzzy implication, and an analysis of fuzzy reasoning mechanisms.

This paper is divided into two parts. The analysis of structural parameters of the FLC is addressed in Part I. In addition, Part I contains five more sections. A brief summary of some of the relevant concepts in fuzzy set theory and fuzzy logic is presented in Section II. The main idea of the FLC is described in Section III, while Section IV describes the fuzzification strategies. In Section V, we discuss the construction of the data base of an FLC. The rule base in Section VI explains the derivation of fuzzy control rules and rule-modification techniques.

Part II consists of four sections. Section I is devoted to the basic aspects of the FLC decision-making logic. Several issues including the definitions of a fuzzy implication,

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TABLE I

1972	Zadeh	A rationale for fuzzy control [145]
1973	Zadeh	Linguistic approach [146]
1974	Mamdani & Assilian	Steam engine control [64]
1976	Rutherford *et al.*	Analysis of control algorithms [5], [7]
1977	Ostergaard	Heat exchanger and cement kiln control [80]
1977	Willaeys *et al.*	Optimal fuzzy control [121]
1979	Komolov *et al.*	Finite automaton [57]
1980	Tong *et al.*	Wastewater treatment process [113]
1980	Fukami, Mizumoto and Tanaka	Fuzzy conditional inference [24]
1983	Hirota and Pedrycz	Probabilistic fuzzy sets (control) [33]
1983	Takagi and Sugeno	Derivation of fuzzy control rules [103]
1983	Yasunobu, Miyamoto *et al.*	Predictive fuzzy control [135]
1984	Sugeno and Murakami	Parking control of a model car [97]
1985	Kiszka, Gupta *et al.*	Fuzzy system stability [55]
1985	Togai and Watanabe	Fuzzy chip [107]
1986	Yamakawa	Fuzzy controller hardware system [130]
1988	Dubois and Prade	Approximate reasoning [21]

compositional operators, the interpretations of sentence connectives "and" and "also," and fuzzy inference mechanisms, are investigated. Section II discusses the defuzzification strategies. Some of the representative applications of the FLC, from laboratory level to industrial process control, are briefly reported in Section III. Finally, we describe some unsolved problems and discuss further challenges in this field.

II. Fuzzy Sets and Fuzzy Logic

For the convenience of the reader, we shall briefly summarize some of the basic concepts of fuzzy set theory and fuzzy logic which will be needed in this paper. A more detailed discussion may be found in [141], [41], [42], [148], [149] and [21].

A. Fuzzy Sets and Terminology

Let U be a collection of objects denoted generically by $\{u\}$, which could be discrete or continuous. U is called the universe of discourse and u represents the generic element of U.

Definition 1: Fuzzy Set: A fuzzy set F in a universe of discourse U is characterized by a membership function μ_F which takes values in the interval $[0,1]$ namely, μ_F: $U \to [0,1]$. A fuzzy set may be viewed as a generalization of the concept of an ordinary set whose membership function only takes two values $\{0,1\}$. Thus a fuzzy set F in U may be represented as a set of ordered pairs of a generic element u and its grade of membership function: $F = \{(u, \mu_F(u)) | u \in U\}$. When U is continuous, a fuzzy set F can be written concisely as $F = \int_U \mu_F(u)/u$. When U is discrete, a fuzzy set F is represented as

$$F = \sum_{i=1}^{n} \mu_F(u_i)/u_i.$$

Definition 2: Support, Crossover Point, and Fuzzy Singleton: The support of a fuzzy set F is the crisp set of all points u in U such that $\mu_F(u) > 0$. In particular, the element u in U at which $\mu_F = 0.5$, is called the crossover point and a fuzzy set whose support is a single point in U with $\mu_F = 1.0$ is referred to as fuzzy singleton.

B. Set Theoretic Operations

Let A and B be two fuzzy sets in U with membership functions μ_A and μ_B, respectively. The set theoretic operations of union, intersection and complement for fuzzy sets are defined via their membership functions. More specifically, see the following.

Definition 3: Union: The membership function $\mu_{A \cup B}$ of the union $A \cup B$ is pointwise defined for all $u \in U$ by

$$\mu_{A \cup B}(u) = \max\{\mu_A(u), \mu_B(u)\}.$$

Definition 4: Intersection: The membership function $\mu_{A \cap B}$ of the intersection $A \cap B$ is pointwise defined for all $u \in U$ by

$$\mu_{A \cap B}(u) = \min\{\mu_A(u), \mu_B(u)\}.$$

Definition 5: Complement: The membership function $\mu_{\bar{A}}$ of the complement of a fuzzy set A is pointwise defined for all $u \in U$ by

$$\mu_{\bar{A}}(u) = 1 - \mu_A(u).$$

Definition 6: Cartesian Product: If $A_1, \cdots, A_n$ are fuzzy sets in $U_1, \cdots, U_n$, respectively, the Cartesian product of $A_1, \cdots, A_n$ is a fuzzy set in the product space $U_1 \times \cdots \times U_n$ with the membership function

$$\mu_{A_1 \times \cdots \times A_n}(u_1, u_2, \cdots u_n) = \min\{\mu_{A_1}(u_1), \cdots, \mu_{A_n}(u_n)\}$$

or

$$\mu_{A_1 \times \cdots \times A_n}(u_1, u_2, \cdots, u_n) = \mu_{A_1}(u_1) \cdot \mu_{A_2}(u_2) \cdots \mu_{A_n}(u_n).$$

Definition 7: Fuzzy Relation: An n-ary fuzzy relation is a fuzzy set in $U_1 \times \cdots \times U_n$ and is expressed as

$$R_{U_1 \times \cdots \times U_n} = \{((u_1, \cdots, u_n), \mu_R(u_1, \cdots, u_n)) | (u_1, \cdots, u_n) \in U_1 \times \cdots \times U_n\}.$$

Definition 8: Sup-Star Composition: If R and S are fuzzy relations in $U \times V$ and $V \times W$, respectively, the composition of R and S is a fuzzy relation denoted by

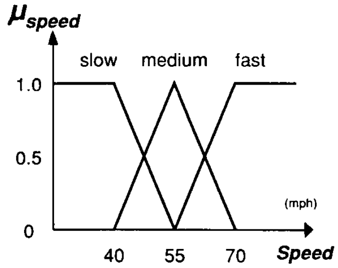

Fig. 1. Diagrammatic representation of fuzzy speeds. "Speed" is linguistic variable with three terms: "slow," "medium," and "high."

$R \circ S$ and is defined by

$$R \circ S = \left\{ \left[(u,w), \sup_{v} (\mu_R(u,v) * \mu_s(v,w)) \right], \quad u \in U, v \in V, w \in W \right\}$$

where $*$ could be any operator in the class of triangular norms, namely, minimum, algebraic product, bounded product, or drastic product (also see Part II [150]).

C. Linguistic Variables and Fuzzy Sets

Definition 9: Fuzzy Number: A fuzzy number F in a continuous universe U, e.g., a real line, is a fuzzy set F in U which is normal and convex, i.e.,

$$\max_{u \in U} \mu_F(u) = 1, \qquad \text{(normal)}$$

$$\mu_F(\lambda u_1 + (1-\lambda) u_2) \geqslant \min(\mu_F(u_1), \mu_F(u_2)), \qquad \text{(convex)}$$
$$u_1, u_1 \in U, \quad \lambda \in [0,1].$$

The use of fuzzy sets provides a basis for a systematic way for the manipulation of vague and imprecise concepts. In particular, we can employ fuzzy sets to represent linguistic variables. A linguistic variable can be regarded either as a variable whose value is a fuzzy number or as a variable whose values are defined in linguistic terms. More specifically: see the following.

Definition 10: Linguistic Variables: A linguistic variable is characterized by a quintuple $(x, T(x), U, G, M)$ in which x is the name of variable; $T(x)$ is the term set of x, that is, the set of names of linguistic values of x with each value being a fuzzy number defined on U; G is a syntactic rule for generating the names of values of x; and M is a semantic rule for associating with each value its meaning. For example, if *speed* is interpreted as a linguistic variable, then its term set T(speed) could be

$$T(\text{speed}) = \{\text{slow, moderate, fast, very slow, more or less fast}, \cdots\}$$

where each term in T(speed) is characterized by a fuzzy set in a universe of discourse $U = [0,100]$. We might interpret "slow" as "a speed below about 40 mph," "moderate" as "a speed close to 55 mph," and "fast" as "a speed above about 70 mph." These terms can be characterized as fuzzy sets whose membership functions are shown in Fig. 1.

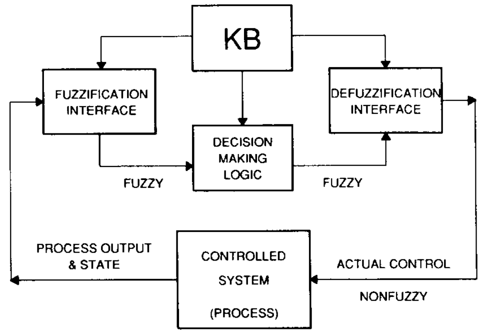

Fig. 2. Basic configuration of fuzzy logic controller (FLC).

D. Fuzzy Logic and Approximate Reasoning

In fuzzy logic and approximate reasoning, there are two important fuzzy implication inference rules named the generalized modus ponens (GMP) and the generalized modus tollens (GMT):

premise 1: x is A',
premise 2: if x is A then y is B,
consequence: y is B' (GMP)

premise 1: y is B′,
premise 2: if x is A then y is B,
consequence: x is A'. (GMT)

The fuzzy implication inference is based on the compositional rule of inference for approximate reasoning suggested by Zadeh in 1973 [146]. Here we introduce fuzzy sets A, A', B, B' via linguistic variables x, y instead of crisp sets in the traditional logic. The GMP, which reduces to "modus ponens" when $A' = A$ and $B' = B$, is closely related to the forward data-driven inference which is particularly useful in the FLC. The GMT, which reduces to "modus tollens" when $B' =$ not B and $A' =$ not A, is closely related to the backward goal-driven inference which is commonly used in expert systems, especially in the realm of medical diagnosis.

Definition 11: Sup-Star Compositional Rule of Inference: If R is a fuzzy relation in $U \times V$, and x is a fuzzy set in U, then the "sup-star compositional rule of inference" asserts that the fuzzy set y in V induced by x is given by [144]

$$y = x \circ R$$

where $x \circ R$ is the sup-star composition of x and R. If the star represents the minimum operator, then this definition reduces to Zadeh's compositional rule of inference [146].

III. Main Idea of the FLC

In this section, we present the main ideas underlying the FLC. To highlight the issues involved, Fig. 2 shows the basic configuration of an FLC, which comprises four principal components: a *fuzzification interface*, a knowledge base, *decision-making logic*, and a *defuzzification interface*.

1) The fuzzification interface involves the following functions:
   a) measures the values of input variables,
   b) performs a scale mapping that transfers the range of values of input variables into corresponding universes of discourse,
   c) performs the function of fuzzification that converts input data into suitable linguistic values which may be viewed as labels of fuzzy sets.
2) The knowledge base comprises a knowledge of the application domain and the attendant control goals. It consists of a "data base" and a "linguistic (fuzzy) control rule base:"
   a) the data base provides necessary definitions, which are used to define linguistic control rules and fuzzy data manipulation in an FLC,
   b) the rule base characterizes the control goals and control policy of the domain experts by means of a set of linguistic control rules.
3) The decisionmaking logic is the kernel of an FLC; it has the capability of simulating human decision-making based on fuzzy concepts and of inferring fuzzy control actions employing fuzzy implication and the rules of inference in fuzzy logic.
4) The defuzzification interface performs the following functions:
   a) a scale mapping, which converts the range of values of output variables into corresponding universes of discourse,
   b) defuzzification, which yields a nonfuzzy control action from an inferred fuzzy control action.

We are now ready to describe the main ideas underlying the FLC in terms of fuzzy logic. The structural parameters involved in the design of an FLC will be discussed at a later point.

A. Fuzzy Conditional Statements and Fuzzy Control Rules

In an FLC, the dynamic behavior of a fuzzy system is characterized by a set of linguistic description rules based on expert knowledge. The expert knowledge is usually of the form

IF (a set of conditions are satisfied) THEN (a set of consequences can be inferred).

Since the antecedents and the consequents of these IF–THEN rules are associated with fuzzy concepts (linguistic terms), they are often called *fuzzy conditional statements*. In our terminology, a *fuzzy control rule* is a fuzzy conditional statement in which the antecedent is a condition in its application domain and the consequent is a control action for the system under control. Basically, fuzzy control rules provide a convenient way for expressing control policy and domain knowledge. Furthermore, several linguistic variables might be involved in the antecedents and the conclusions of these rules. When this is the case, the system will be referred to as a multi-input–multi-output (MIMO) fuzzy system. For example, in the case of two-input–single-output (MISO) fuzzy systems, fuzzy control rules have the form:

$$
\begin{aligned}
&R_1\text{: if } x \text{ is } A_1 \text{ and } y \text{ is } B_1 \text{ then } z \text{ is } C_1,\\
&R_2\text{: if } x \text{ is } A_2 \text{ and } y \text{ is } B_2 \text{ then } z \text{ is } C_2,\\
&\quad \cdots \quad \cdots \quad \cdots \quad \cdots\\
&\quad \cdots \quad \cdots \quad \cdots \quad \cdots\\
&R_n\text{: if } x \text{ is } A_n \text{ and } y \text{ is } B_n \text{ then } z \text{ is } C_n,
\end{aligned}
$$

where x, y, and z are linguistic variables representing two process state variables and one control variable; A_i, B_i, and C_i are linguistic values of the linguistic variables x, y, and z in the universes of discourse U, V, and W, respectively, with $i = 1, 2, \cdots, n$; and an implicit sentence connective also links the rules into a rule set or, equivalently, a rule base.

A fuzzy control rule, such as "if (x is A_i and y is B_i) then (z is C_i)," is implemented by a *fuzzy implication* (fuzzy relation) R_i and is defined as follows:

$$
\begin{aligned}
\mu_{R_i} &\triangleq \mu_{(A_i \text{ and } B_i \to C_i)}(u, v, w)\\
&= \left[\mu_{A_i}(u) \text{ and } \mu_{B_i}(v)\right] \to \mu_{C_i}(w)
\end{aligned}
$$

where A_i and B_i is a fuzzy set $A_i \times B_i$ in $U \times V$; $R_i \triangleq (A_i \text{ and } B_i) \to C_i$ is a fuzzy implication (relation) in $U \times V \times W$; and $\to$ denotes a fuzzy implication function. As will be seen later, there are many ways in which a fuzzy implication may be defined.

B. Fuzzification Operator

A fuzzification operator has the effect of transforming crisp data into fuzzy sets. Symbolically,

$$x = \mathit{fuzzifier}(x_0)$$

where x_0 is a crisp input value from a process; x is a fuzzy set; and *fuzzifier* represents a fuzzification operator.

C. Sentence Connective Operators

An FLC consists of a set of fuzzy control rules which are related by the dual concepts of fuzzy implication and the sup-star compositional rule of inference. These fuzzy control rules are combined by using the sentence connectives *and* and *also*. Since each fuzzy control rule is represented by a fuzzy relation, the overall behavior of a fuzzy system is characterized by these fuzzy relations. In other words, a fuzzy system can be characterized by a single fuzzy relation which is the combination of the fuzzy relations in the rule set. The combination in question involves the sentence connective *also*. Symbolically,

$$R = \mathit{also}(R_1, R_2, \cdots, R_i, \cdots, R_n)$$

where *also* represents a sentence connective.

D. Compositional Operator

To infer the output z from the given process states x, y and the fuzzy relation R, the sup-star compositional rule of inference is applied

$$z = y \circ (x \circ R)$$

where $\circ$ is the sup-star composition.

E. Defuzzification Operator

The output of the inference process so far is a fuzzy set, specifying a possibility distribution of control action. In the on-line control, a nonfuzzy (crisp) control action is usually required. Consequently, one must defuzzify the fuzzy control action (output) inferred from the fuzzy control algorithm, namely:

$$z_0 = \mathit{defuzzifier}(z),$$

where z_0 is the nonfuzzy control output and *defuzzifier* is the defuzzification operator.

F. Design Parameters of the FLC

The principal design parameters for an FLC are the following:

1) fuzzification strategies and the interpretation of a fuzzification operator (fuzzifier),
2) data base:
   a) discretization/normalization of universes of discourse,
   b) fuzzy partition of the input and output spaces,
   c) completeness,
   d) choice of the membership function of a primary fuzzy set;
3) rule base:
   a) choice of process state (input) variables and control (output) variables of fuzzy control rules,
   b) source and derivation of fuzzy control rules,
   c) types of fuzzy control rules,
   d) consistency, interactivity, completeness of fuzzy control rules;
4) decision making logic:
   a) definition of a fuzzy implication,
   b) interpretation of the sentence connective *and*,
   c) interpretation of the sentence connective *also*,
   d) definitions of a compositional operator,
   e) inference mechanism;
5) defuzzification strategies and the interpretation of a defuzzification operator (defuzzifier).

IV. Fuzzification Strategies

Fuzzification is related to the vagueness and imprecision in a natural language. It is a subjective valuation which transforms a measurement into a valuation of a subjective value, and hence it could be defined as a mapping from an observed input space to fuzzy sets in certain input universes of discourse. Fuzzification plays an important role in dealing with uncertain information which might be objective or subjective in nature.

In fuzzy control applications, the observed data are usually crisp. Since the data manipulation in an FLC is based on fuzzy set theory, fuzzification is necessary during an earlier stage. Experience with the design of an FLC suggests the following principal ways of dealing with fuzzification.

1) A fuzzification operator "conceptually" converts a crisp value into a fuzzy singleton within a certain universe of discourse. Basically, a fuzzy singleton is a precise value and hence no fuzziness is introduced by fuzzification in this case. This strategy has been widely used in fuzzy control applications since it is natural and easy to implement. It interprets an input x_0 as a fuzzy set A with the membership function $\mu_A(x)$ equal to zero except at the point x_0, at which $\mu_A(x_0)$ equals one.
2) Observed data are disturbed by random noise. In this case, a fuzzification operator should convert the probabilistic data into fuzzy numbers, i.e., fuzzy (possibilistic) data. In this way, computational efficiency is enhanced since fuzzy numbers are much easier to manipulate than random variables. In [76], an isosceles triangle was chosen to be the fuzzification function. The vertex of this triangle corresponds to the mean value of a data set, while the base is twice the standard deviation of the data set. In this way, we form a triangular fuzzy number which is convenient to manipulate [42]. In this connection, it should be noted that Dubois and Prade [20] defined a bijective transformation which transforms a probability measure into a possibility measure by using the concept of the degree of necessity. Basically, the necessity of an event, E, is the added probability of elementary events in E over the probability assigned to the most frequent elementary event outside of E. Based on the method of Dubois and Prade, the histogram of the measured data may be used to estimate the membership function for the transformation of probability into possibility [17].
3) In large scale systems and other applications, some observations relating to the behavior of such systems are precise, while others are measurable only in a statistical sense, and some, referred to as "hybrids," require both probabilistic and possibilistic modes of characterization. The strategy of fuzzification in this case is to use the concept of "hybrid numbers" [42], which involve both uncertainty (fuzzy numbers) and randomness (random numbers). The use of hybrid number arithmetic in the design of an FLC suggests a promising direction that is in need of further exploration.

V. Data Base

The knowledge base of an FLC is comprised of two components, namely, a data base and a fuzzy control rule base. We shall address some issues relating to the data

TABLE II
QUANTIZATION AND PRIMARY FUZZY SETS USING A NUMERICAL DEFINITION

Level No.	Range	*NB*	*NM*	*NS*	*ZE*	*PS*	*PM*	*PM*
−6	$x_0 \leqslant -3.2$	1.0	0.3	0.0	0.0	0.0	0.0	0.0
−5	$-3.2 < x_0 \leqslant -1.6$	0.7	0.7	0.0	0.0	0.0	0.0	0.0
−4	$-1.6 < x_0 \leqslant -0.8$	0.3	1.0	0.3	0.0	0.0	0.0	0.0
−3	$-0.8 < x_0 \leqslant -0.4$	0.0	0.7	0.7	0.0	0.0	0.0	0.0
−2	$-0.4 < x_0 \leqslant -0.2$	0.0	0.3	1.0	0.3	0.0	0.0	0.0
−1	$-0.2 < x_0 \leqslant -0.1$	0.0	0.0	0.7	0.7	0.0	0.0	0.0
0	$-0.1 < x_0 \leqslant +0.1$	0.0	0.0	0.3	1.0	0.3	0.0	0.0
1	$+0.1 < x_0 \leqslant +0.2$	0.0	0.0	0.0	0.7	0.7	0.0	0.0
2	$+0.2 < x_0 \leqslant +0.4$	0.0	0.0	0.0	0.3	1.0	0.3	0.0
3	$+0.4 < x_0 \leqslant +0.8$	0.0	0.0	0.0	0.0	0.7	0.7	0.0
4	$+0.8 < x_0 \leqslant +1.6$	0.0	0.0	0.0	0.0	0.3	1.0	0.3
5	$+1.6 < s_0 \leqslant +3.2$	0.0	0.0	0.0	0.0	0.0	0.7	0.7
6	$3.2 \leqslant x_0$	0.0	0.0	0.0	0.0	0.0	0.3	1.0

base in this section and to the rule base in the next section. The concepts associated with a data base are used to characterize fuzzy control rules and fuzzy data manipulation in an FLC. These concepts are subjectively defined and based on experience and engineering judgment. In this connection, it should be noted that the correct choice of the membership functions of a term set plays an essential role in the success of an application. In what follows, we shall discuss some of the important aspects relating to the construction of the data base in an FLC.

A. Discretization / Normalization of Universes of Discourse

The representation of uncertain information with fuzzy sets brings up the problem of quantifying such information for digital computer processing. In general, the representation depends on the nature of the universe of discourse. A universe of discourse in an FLC is either discrete or continuous. If the universe is continuous, a discrete universe may be formed by a discretization of the continuous universe. Furthermore, a continuous universe may be normalized, as will be seen at a later point in this section.

1) Discretization of a Universe of Discourse: Discretization of a universe of discourse is frequently referred to as quantization. In effect, quantization discretizes a universe into a certain number of segments (quantization levels). Each segment is labeled as a generic element, and forms a discrete universe. A fuzzy set is then defined by assigning grade of membership values to each generic element of the new discrete universe. A look-up table based on discrete universes, which defines the output of a controller for all possible combinations of the input signals, can be implemented by off-line processing in order to shorten the running time of the controller [90]. In the case of an FLC with continuous universes, the number of quantization levels should be large enough to provide an adequate approximation and yet be small to save memory storage. The choice of quantization levels has an essential influence on how fine a control can be obtained. For example, if a universe is quantized for every five units of measurement instead of ten units, then the controller is twice as sensitive to the observed variables.

For the purpose of discretization, we need a scale mapping, which serves to transform measured variables into values in the discretized universe. The mapping can be uniform (linear), nonuniform (nonlinear), or both. The choice of quantization levels reflects some *a priori* knowledge. For example, coarse resolution could be used for large errors and fine resolution for small errors. Thus, in a three-input-one-output fuzzy system, we may have control rules of the form:

$$R_i\text{: if error } (e) \text{ is } A_i, \text{ sum of errors } (ie) \text{ is } B_i,$$
$$\text{and change of error } (de) \text{ is } C_i \text{ then output is } D_i.$$

A simple instance of an FLC can be represented by

$$K_4[u(k)] = F[K_1e(k), K_2ie(k), K_3de(k)],$$

where F denotes the fuzzy relation defined by the rule base and $K_i, i = 1,2,3,4$, represents an appropriate scaling mapping. In this relation, we see an analogy to the parameters of a conventional PID controller [63],[105], in which as a special case F is a linear function of its arguments. An example of discretization is shown in Table II, where a universe of discourse is discretized into 13 levels with seven terms (*primary fuzzy sets*) defined on it. In general, due to discretization, the peformance of an FLC is less sensitive to small deviations in the values of the process state variables.

2) Normalization of a Universe of Discourse: The normalization of a universe requires a discretization of the universe of discourse into a finite number of segments, with each segment mapped into a suitable segment of the normalized universe. In this setting a fuzzy set is then defined by assigning an explicit function to its membership function. The normalization of a continuous universe also involves *a priori* knowledge of the input/output space. The scale mapping can be uniform, nonuniform, or both. One example is shown in Table III, where the universe of discourse, $[-6.0, +4.5]$, is transformed into the normalized closed interval $[-1, +1]$.

B. Fuzzy Partition of Input and Output Spaces

A linguistic variable in the antecedent of a fuzzy control rule forms a fuzzy input space with respect to a certain universe of discourse, while that in the consequent

TABLE III
NORMALIZATION AND PRIMARY FUZZY SETS USING A FUNCTIONAL DEFINITION

Normalized Universe	Normalized Segments	Range	u_f	σ_f	Primary Fuzzy Sets
	$[-1.0, -0.5]$	$[-6.9, -4.1]$	-1.0	0.4	*NB*
	$[-0.5, -0.3]$	$[-4.1, -2.2]$	-0.5	0.2	*NM*
	$[-0.3, -0.0]$	$[-2.2, -0.0$	-0.2	0.2	*NM*
$[-1.0, +1.0]$	$[-0.0, +0.2]$	$[-0.0, +1.0]$	0.0	0.2	*ZE*
	$[+0.2, +0.6]$	$[+1.0, +2.5]$	0.2	0.2	*PS*
	$[+0.6, +1.0]$	$[+2.5, +4.5]$	0.5	0.2	*PM*
			1.0	0.4	*PB*

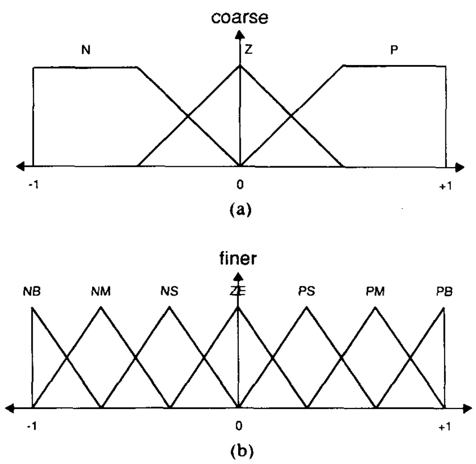

Fig. 3. Diagrammatic representation of fuzzy partition. (a) Coarse fuzzy partition with three terms: *N*, negative; *ZE*, zero; and *P*, positive. (b) Finer fuzzy partition with seven terms: *NB*, negative big; *NM*, negative medium; *NS*, negative small; *ZE*, zero; *PS*, positive small; *PM*, positive medium; and *PB*, positive big.

of the rule forms a fuzzy output space. In general, a linguistic variable is associated with a term set, with each term in the term set defined on the same universe of discourse. A fuzzy partition, then, determines how many terms should exist in a term set. This is equivalent to finding the number of primary fuzzy sets. The number of primary fuzzy sets determines the granularity of the control obtainable with an FLC. The primary fuzzy sets (linguistic terms) usually have a meaning, such as *NB*: negative big; *NM*: negative medium; *NS*: negative small; *ZE*: zero; *PS*: positive small; *PM*: positive medium; and *PB*: positive big. A typical example is shown in Fig. 3, depicting two fuzzy partitions in the same normalized universe $[-1, +1]$. Membership functions having the forms of triangle-shaped and trapezoid-shaped functions are used here. Since a normalized universe implies the knowledge of the input/output space via appropriate scale mappings, a well-formed term set can be achieved as shown. If this is not the case, or a nonnormalized universe is used, the terms could be asymmetrical and unevenly distributed in the universe. Furthermore, the cardinality of a term set in a fuzzy input space determines the maximum number of fuzzy control rules that we can construct. In the case of two-input-one-output fuzzy systems, if the cardinalities of $T(x)$ and $T(y)$ are 3 and 7, respectively, the maximum rule number is 3×7. It should be noted that the fuzzy partition of the fuzzy input/output space is not deterministic and has no unique solution. A heuristic cut and trial procedure is usually needed to find the optimal fuzzy partition.

C. Completeness

Intuitively, a fuzzy control algorithm should always be able to infer a proper control action for every state of process. This property is called "completeness." The completeness of an FLC relates to its data base, rule base, or both.

1) Data Base Strategy: The data base strategy is concerned with the supports on which primary fuzzy sets are defined. The union of these supports should cover the related universe of discourse in relation to some level set ϵ. This property of an FLC is called ϵ-completeness. In general, we choose the level ϵ at the crossover point as shown in Fig. 3, implying that we have a strong belief in the positive sense of the fuzzy control rules which are associated with the FLC. In this sense, a dominant rule always exists and is associated with the degree of belief greater than 0.5. In the extreme case, two dominant rules are activated with equal belief 0.5.

2) Rule Base Strategy: The rule base strategy has to do with the fuzzy control rules themselves. The property of completeness is incorporated into fuzzy control rules through design experience and engineering knowledge. An additional rule is added whenever a fuzzy condition is not included in the rule base, or whenever the degree of partial match between some inputs and the predefined fuzzy conditions is lower than some level, say 0.5. The former shows that no control action will result. The latter indicates that no dominant rule will be fired.

D. Membership Function of a Primary Fuzzy Set

There are two methods used for defining fuzzy sets, depending on whether the universe of discourse is discrete or continuous: a) numerical and b) functional.

1) Numerical Definition: In this case, the grade of membership function of a fuzzy set is represented as a vector of numbers whose dimension depends on the degree of discretization. An illustrative example is shown in Table II. In this case, the membership function of each primary

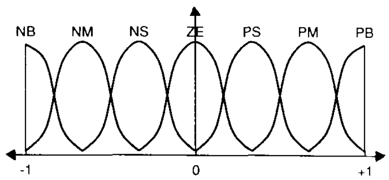

Fig. 4. Example of functional definition of primary fuzzy sets.

fuzzy set has the form of

$$\mu_f(u) = \sum_{i=1}^{5} a_i / u_i,$$

where

$$a = [0.3, 0.7, 1.0, 0.7, 0.3].$$

2) Functional Definition: A functional definition expresses the membership function of a fuzzy set in a functional form, typically a bell-shaped function, triangle-shaped function, trapezoid-shaped function, etc. Such functions are used in FLC because they lead themselves to manipulation through the use of fuzzy arithmetic. The functional definition can readily be adapted to a change in the normalization of a universe. Table III and Fig. 4 show an example of a functional definition expressed as:

$$\mu_f(x) = \exp\left\{\frac{-(x-u_f)^2}{2\sigma_f^2}\right\}.$$

Note that if the normalized universe is changed, the parameters u_f, σ_f should be changed accordingly.

Either a numerical definition or functional definition may be used to assign the grades of membership to the primary fuzzy sets. The choice of grades of membership is based on the subjective criteria of the decision. In particular, as we mentioned before, if the measurable data might be disturbed by noise, the membership functions should be sufficiently wide to reduce the sensitivity to noise. This raises the issue of the fuzziness or, more accurately, the specificity of a membership function, which affects the robustness of an FLC. This issue is discussed in greater detail in [58].

VI. Rule Base

A fuzzy system is characterized by a set of linguistic statements based on expert knowledge. The expert knowledge is usually in the form of "if–then" rules, which are easily implemented by fuzzy conditional statements in fuzzy logic. The collection of fuzzy control rules that are expressed as fuzzy conditional statements forms the rule base or the rule set of an FLC. In this section, we shall examine the following topics related to fuzzy control rules: choice of process state (input) variables and control (output) variables, source and derivation, justification, types of fuzzy control rules, and properties of consistency, interactivity, and completeness.

A. Choice of Process State Variables and Control Variables of Fuzzy Control Rules

Fuzzy control rules are more conveniently formulated in linguistic rather than numerical terms. The proper choice of process state variables and control variables is essential to the characterization of the operation of a fuzzy system. Furthermore, the selection of the linguistic variables has a substantial effect on the performance of an FLC. As was stated earlier, experience and engineering knowledge play an important role during this selection stage. In particular, the choice of linguistic variables and their membership function have a strong influence on the linguistic structure of an FLC. Typically, the linguistic variables in an FLC are the state, state error, state error derivative, state error integral, etc.

B. Source and Derivation of Fuzzy Control Rules

There are four modes of derivation of fuzzy control rules, as reported in [103]. These four modes are not mutually exclusive, and it seems likely that a combination of them would be necessary to construct an effective method for the derivation of fuzzy control rules.

1) Expert Experience and Control Engineering Knowledge: Fuzzy control rules have the form of fuzzy conditional statements that relate the state variables in the antecedent and process control variables in the consequents. In this connection, it should be noted that in our daily life most of the information on which our decisions are based is linguistic rather than numerical in nature. Seen in this perspective, fuzzy control rules provide a natural framework for the characterization of human behavior and decisions analysis. Many experts have found that fuzzy control rules provide a convenient way to express their domain knowledge. This explains why most FLCs are based on the knowledge and experience which are expressed in the language of fuzzy if–then rules [64], [47], [50], [80], [82], [59], [118], [113], [58], [127], [4].

The formulation of fuzzy control rules can be achieved by means of two heuristic approaches. The most common one involves an introspective verbalization of human expertise. A typical example of such verbalization is the operating manual for a cement kiln. Another approach includes an interrogation of experienced experts or operators using a carefully organized questionnaire. In this manner, we can form a prototype of fuzzy control rules for a practicular application domain. For optimized performance, the use of cut and trial procedures is usually a necessity.

2) Based on Operator's Control Actions: In many industrial man–machine control systems, the input–output relations are not known with sufficient precision to make it possible to employ classical control theory for modeling and simulation. And yet skilled human operators can control such systems quite successfully without having any quantitative models in mind. In effect, a human operator employs–consciously or subconsciously—a set of fuzzy

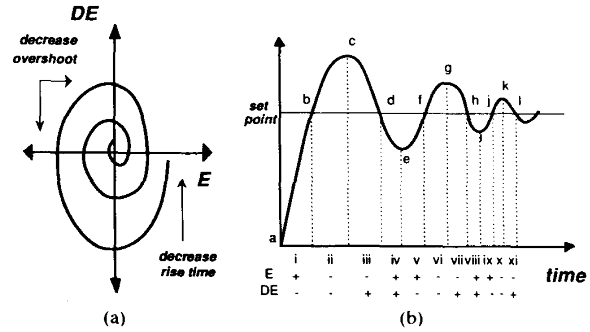

Fig. 5. Rule justification by using phase plane. (a) Phase-plane trajectory. (b) System step response.

if-then rules to control the process. As was pointed out by Sugeno, to automate such processes, it is expedient to express the operator's control rules as fuzzy if-then rules employing linguistic variables. In practice, such rules can be deduced from the observation of human controller's actions in terms of the input-output operating data [97]-[99].

3) Based on the Fuzzy Model of a Process: In the linguistic approach, the linguistic description of the dynamic characteristics of a controlled process may be viewed as a fuzzy model of the process. Based on the fuzzy model, we can generate a set of fuzzy control rules for attaining optimal performance of a dynamic system. The set of fuzzy control rules forms the rule base of an FLC. Although this approach is somewhat more complicated, it yields better performance and reliability, and provides a more tractable structure for dealing theoretically with the FLC. However, this approach to the design of an FLC has not as yet been fully developed.

4) Based on Learning: Many FLCs have been built to emulate human decision-making behavior, but few are focused on human learning, namely, the ability to create fuzzy control rules and to modify them based on experience. Procyk and Mamdani [87] described the first self-organizing controller (SOC). The SOC has a hierarchical structure which consists of two rule bases. The first one is the general rule base of an FLC. The second one is constructed by "meta-rules" which exhibit human-like learning ability to create and modify the general rule base based on the desired overall performance of the system. Recently, further studies relating to the SOC have been carried out at Queen Mary College and elsewhere [60], [94], [102], [95], [106]. A very interesting example of a fuzzy rule-based system which has a learning capability is Sugeno's fuzzy car [97], [99]. Sugeno's fuzzy car can be trained to park by itself.

C. Justification of Fuzzy Control Rules

There are two principal approaches to the derivation of fuzzy control rules. The first is a heuristic method in which a collection of fuzzy control rules is formed by analyzing the behavior of a controlled process. The control rules are derived in such a way that the deviation from a desired state can be corrected and the control objective can be achieved. The derivation is purely heuristic in nature and relies on the qualitative knowledge of process behavior. Several methods of adjustment of rule selection have been studied [1], [49], [7], [6]. A brief review of these results is given in the following. The second approach is basically a deterministic method which can systematically determine the linguistic structure and/or parameters of the fuzzy control rules that satisfy the control objectives and constraints [111], [103], [104], [101].

Mamdani [1] proposed a prescriptive algorithm for deriving the "best" control rules by restricting system responses to a "prescriptive fuzzy band" which is specified by fuzzy control rules. However, the convergence of the prescriptive method requires a careful analysis.

King and Mamdani [49] introduced another useful method for rule justification. So-called "scale mappings" should be adjusted first so that the system trajectory can terminate on a desired state. The rule justification is done by referring to a closed system trajectory in a phase plane. A knowledge of parameter-adjusting based on phase plane analysis (e.g., overshoot, rise time) and an intuitive feel for the behavior of the closed loop system are required. The principle of global rule modification in symmetry and monotonicity is also employed.

For example, Fig. 5 shows the system response of a process to be controlled, where the input variables of the FLC are the error (E) and error derivative (DE). The output is the change of the process input (CI). We assume that the term sets of input/output variables have the same cardinality, 3, with a common term {negative, zero, positive}. The prototype of fuzzy control rules is tabulated in Table IV and a justification of fuzzy control rules is added in Table V. The corresponding rule of region i can be formulated as rule R_i and has the effect of shortening the rise time. Rule R_{ii} for region ii decreases the overshoot of the system's response. More specifically,

R_i: if (E is positive and DE is negative) then CI is positive,

R_{ii}: if (E is negative and DE is negative) then CI is negative.

TABLE IV
PROTOTYPE OF FUZZY CONTROL RULES WITH TERM SETS
{NEGATIVE, ZERO, POSITIVE}

Rule No.	*E*	*DE*	*CI*	Reference Point
1	*P*	*Z*	*P*	a, e, i
2	*Z*	*N*	*N*	b, f, j
3	*N*	*Z*	*N*	c, g, k
4	*Z*	*P*	*P*	d, h, l
5	*Z*	*Z*	*Z*	set point

TABLE V
RULE JUSTIFICATION WITH TERM SETS
{NEGATIVE, ZERO, POSITIVE}

Rule No.	*E*	*DE*	*CI*	Reference Range
6	*P*	*N*	*P*	i (rise time), v
7	*N*	*N*	*N*	ii (overshoot), vi
8	*N*	*P*	*N*	iii, vii
9	*P*	*P*	*P*	iv, viii
10	*P*	*N*	*Z*	ix
11	*N*	*P*	*Z*	xi

TABLE VI
PROTOTYPE OF FUZZY CONTROL RULES WITH TERM SETS
{*NB, NM, NS, ZE, PS, PM, PB*}

Rule No.	*E*	*DE*	*CI*	Reference Point
1	*PB*	*ZE*	*PB*	a
2	*PM*	*ZE*	*PM*	e
3	*PS*	*ZE*	*PS*	i
4	*ZE*	*NB*	*NB*	b
5	*ZE*	*NM*	*NM*	f
6	*ZE*	*NS*	*NS*	j
7	*NB*	*ZE*	*NB*	c
8	*NM*	*ZE*	*NM*	g
9	*NS*	*ZE*	*NS*	k
10	*ZE*	*PB*	*PB*	d
11	*ZE*	*PM*	*PM*	h
12	*ZE*	*PS*	*PS*	l
13	*ZE*	*ZE*	*ZE*	set point

TABLE VII
RULE JUSTIFICATION WITH TERM SETS {*NB, NM, NS, ZE, PS, PM, PM*}

Rule No.	*E*	*DE*	*CI*	Reference Range
14	*PB*	*NS*	*PM*	i (rise time)
15	*PS*	*NB*	*NM*	i (overshoot)
16	*NB*	*PS*	*NM*	iii
17	*NS*	*PB*	*PM*	iii
18	*PS*	*NS*	*ZE*	ix
19	*NS*	*PS*	*ZE*	xi

Better control performance can be obtained by using finer fuzzy partitioned subspaces, for example, with the term set {*NB, NM, NS, ZE, PS, PM, PB*}. The prototype and the justification of fuzzy control rules are also given in Table VI and Table VII.

A slightly modified method was suggested in [7]. It tracked the linguistic trajectory of a closed loop system in a "linguistic phase plane." The main idea is that scale mappings should be adjusted first to yield approximately a desired trajectory behavior. This can be inferred from the linguistic trajectories. Then rule modification can be accomplished by using the linguistic trajectory behavior to optimize the system response in the linguistic phase plane.

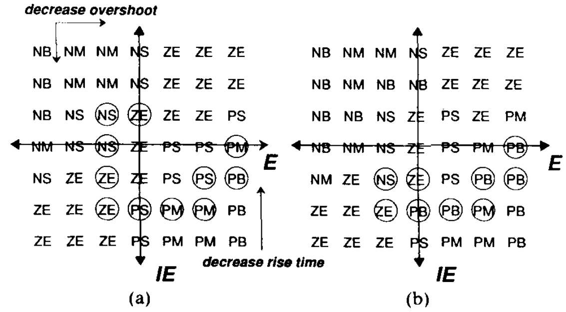

Fig. 6. Rule justification by using a linguistic phase plane. (a) Linguistic trajectory with initial rules. (b) Linguistic trajectory with modified rules. (From Braae and Rutherford [7].

An additional advantage of this approach is that the measurement noise appearing in the linguistic phase plane is less of a problem than that in the nonlinguistic phase plane. An example is shown in Fig. 6.

An approach to generating the rule base of an FLC, which is analogous to the conventional controller design by pole placement, is described in [6]. Braae and Rutherford assumed that the fuzzy control rules of an open system (process) and a desired closed-loop system were initially given. The purpose is to synthesize a linguistic control element (FLC) based on the fuzzy models described above. The main idea is to invert the low order linguistic model of a certain open loop system. However, linguistic inversion mappings are usually incomplete or multivalued. So, an "approximate" strategy, which is somewhat heuristic and subjective, is necessary to complete the inverse mapping which has a reasonable singled-valued solution. This approximation has substantial effect on "linguistic substitution" which further determines a fuzzy controller. This method is restricted to relative low order systems but it provides an explicit solution for rule generation of the FLC, assuming that fuzzy models of the open and closed systems are available.

The systematic rule justification has recently been proposed and studied by means of fuzzy relational equations [13], [15], [84], [125] and linguistic control rules [111], [103], [104], [101]. The basic notion of these two approaches is so-called "fuzzy identification." As in conventional identification, the fuzzy identification comprises two phases, namely, structure identification and parameter estimation. The studies in question deal with one, or both.

Tong [111] introduced the concept of "logical examination" (LE) for converting process input–output data into a set of fuzzy control rules. Tong tackled both identification problems simultaneously, and used a correlation analysis of the LE to determine the linguistic structure. However, it is still somewhat heuristic and subjective, and encounters difficulties in the identification of multivariable fuzzy systems.

Takagi and Sugeno [103] proposed a fuzzy identification algorithm for modeling human operator's control actions. In this case, a suitable linguistic structure is easy to find since one can observe and/or ask for the kind of information which the operator needs, such as process state variables. The fuzzy control rules to be identified have the form of

$$R_i\text{: if } x \text{ is } A_i,\cdots, \text{ and } y \text{ is } B_i \text{ then } z = f_i(x,\cdots,y)$$

where $x,\cdots,y$, and z are linguistic variables representing the process state variables and the control variable; $A_i,\cdots,B_i$ are linguistic terms of the linguistic variables $x,\cdots,y$, and z in the universes of discourse $U,\cdots,V$, and W, respectively, with $i=1,2,\cdots,n$; and z is a logical function of the process state variables such as a linear function of $x,\cdots,y$. In this way, the problem is reduced to parameter estimation, which is done by optimizing a least-square performance index via a weighted linear regression method. The inference mechanism of this FLC will be discussed in Part II [150].

Sugeno has successfully applied this method to the design of an FLC for navigating a model car through a crank-shaped curve [98] and for parking a model car in a garage [97],[99]. Sugeno's method provides a more systematic approach to the design of an FLC, and the experimental results are quite remarkable. However, some steps of this algorithm, such as the choice of process state variables, the fuzzy partition of input spaces, and the choice of the membership functions of primary fuzzy sets, depend on trial-and-error.

Recently, Takagi and Sugeno [104] improved their algorithm so that parameter estimation can be fully implemented. At issue is the problem of structure identification, which is partly addressed in this paper. Further research on this problem has been reported by Sugeno and Kang in [101].

Another approach based on fuzzy relational equations is directed at the same problems. The structure identification requires the determination of the system order and time delays of discrete-time fuzzy models, while the parameter estimation reduces to the determination of the overall fuzzy relation matrix from the input-output data of the system. The reader is referred to [13],[15],[84],[125] for further details.

D. Types of Fuzzy Control Rules

Depending on their nature, two types of fuzzy control rules, state evaluation fuzzy control rules and object evaluation fuzzy control rules, are currently in use in the design of the FLC.

1) State Evaluation Fuzzy Control Rules: Most FLC's have state evaluation fuzzy control rules which, in the case of MISO systems, are characterized as a collection of rules of the form

$$\begin{aligned}
&R_1\text{: if } x \text{ is } A_1,\cdots, \text{ and } y \text{ is } B_1 \text{ then } z \text{ is } C_1\\
&R_2\text{: if } x \text{ is } A_2,\cdots, \text{ and } y \text{ is } B_2 \text{ then } z \text{ is } C_2\\
&\qquad\cdots\ \cdots\ \cdots\ \cdots\\
&\qquad\cdots\ \cdots\ \cdots\ \cdots\\
&R_n\text{: if } x \text{ is } A_n,\cdots, \text{ and } y \text{ is } B_n \text{ then } z \text{ is } C_n
\end{aligned}$$

where $x,\cdots,y$, and z are linguistic variables representing the process state variables and the control variable; $A_i,\cdots,B_i$, and C_i are the linguistic values of the linguistic variables $x,\cdots,y$, and z in the universes of discourse $U,\cdots,V$, and W, respectively, $i=1,2,\cdots,n$.

In a more general version, the consequent is represented as a function of the process state variables $x,\cdots,y$, i.e.,

$$R_i\text{: if } x \text{ is } A_i,\cdots, \text{ and } y \text{ is } B_i \text{ then } z = f_i(x,\cdots,y).$$

Fuzzy control rules of this type, which are referred to as "state evaluation fuzzy control rules," evaluate the process state (e.g., state, state error, state integral) at time t and compute a fuzzy control action at time t as a function of $(x,\cdots,y)$ and the control rules in the rule set.

2) Object Evaluation Fuzzy Control Rules: Yasunobu, Miyamoto, and Ihara [135] proposed another algorithm which predicts present and future control actions and evaluates control objectives. It is called "object evaluation fuzzy control," or "predictive fuzzy control." The rules in question, which are derived from skilled operator's experience, are referred to as "object evaluation fuzzy control rules." A typical rule is described as

$$R_i\text{: if } (u \text{ is } C_i \to (x \text{ is } A_i \text{ and } y \text{ is } B_i)) \text{ then } u \text{ is } C_i.$$

A control command is inferred from an objective evaluation of a fuzzy control result that satisfies the desired states and objectives. A control command u takes a crisp set as a value, and x, y are performance indices for the evaluation of the ith rule, taking values such as "good" or "bad." The most likely control rule is selected through predicting the results (x, y) corresponding to every control command C_i.

In linguistic terms, the rule is interpreted as: "if the performance index x is A_i and index y is B_i when a control command u is chosen to be C_i, then this rule is selected and the control command C_i is taken to be the output of the controller."

In automatic train operation, a typical control rule is *if the control notch is not changed and if the train stops in the predetermined allowance zone, then the control notch is not changed*.

It is well known that systems control encounters difficulties in satisfying multiple performance indices simultaneously and in achieving accurate control in the presence of disturbances. In such circumstances, fuzzy control provides an effective framework for solution. However, the state evaluation fuzzy control does not evaluate the computed control actions as human operators do. By contrast, the predictive fuzzy control provides a mechanism for evaluation so that the desired states and control objectives can be achieved more easily. It should be noted that predictive control has been successfully applied to automatic train operation [135], [136], [139] as well as to automatic container crane operation systems [137]–[139]. Tests have shown that this type of control is capable of operating trains and cranes as skillfully as an experienced operator.

E. Properties of Consistency, Interactivity, and Completeness

1) Completeness: Please refer to Section V of this paper.

2) Number of Fuzzy Control Rules: There is no general procedure for deciding on the optimal number of fuzzy control rules since a number of factors are involved in the decision, e.g., performance of the controller, efficiency of computation, human operator behavior, and the choice of linguistic variables.

3) Consistency of Fuzzy Control Rules: If the derivation of fuzzy control rules is based on the human operator experience, the rules may be subjected to different performance criteria. In practice, it is important to check on the consistency of fuzzy control rules in order to minimize the possibility of contradiction. [64],[12].

4) Interactivity of Fuzzy Control Rules: Assuming that a collection of fuzzy control rules has the form

$$R_i\text{: if } x \text{ is } A_i \text{ then } z \text{ is } C_i, \qquad i = 1, \cdots, n.$$

If an input x_0 is A_i, we would expect that the control action z is C_i. In fact, the control action z may be a subset or a superset of C_i [12], [26], [85], [18], [19], depending on the definition of fuzzy implication and sup-star composition. This may happen as a consequence of interaction between the rules.

The problem of interaction is complex and not as yet well understood. The reported research in [12], [26], [85], [18], [19] indicates that interactivity of rules can be controlled by the choice of fuzzy implication and sup-star composition. The consistency of rules may be improved through the use of the concept of a fuzzy clustering of fuzzy control rules. In this connection, it should be noted that Sugeno's reasoning and identification algorithm provides an alternative solution to these problems [104], [101].

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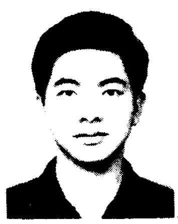

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