Computer Architecture: Fundamentals, Tradeoffs, Challenges

Chapter 11: Floating Point Arithmetic

Yale Patt

The University of Texas at Austin

Austin, Texas

Spring, 2023

Computer Architecture: Fundamentals, Tradeoffs, Challenges

Chapter 11: Floating Point Arithmetic

Yale Patt

The University of Texas at Austin

Austin, Texas

Spring, 2021

Outline

- *Scientific Notation and its representations in binary*
  - *Avogadro’s Number*
  - *Format (Precision vs Range)*
  - *Common floating point data types*
  - *Why it is called floating point*
- *My 6-bit floating point data type*
  - *Redundant most significant bit*
  - *Excess code for exponents (infinity, subnormals)*
- *Rounding*
  - *Four rounding modes*
  - *Guard, Round, Sticky bits*
  - *Wobble*
- *Infinities, NaNs*
- *Subnormal numbers (gradual underflow)*
- *Exceptions: Invalid, DivBy0, Underflow, Overflow, Inexact*
  - *Quiet vs Signaling*

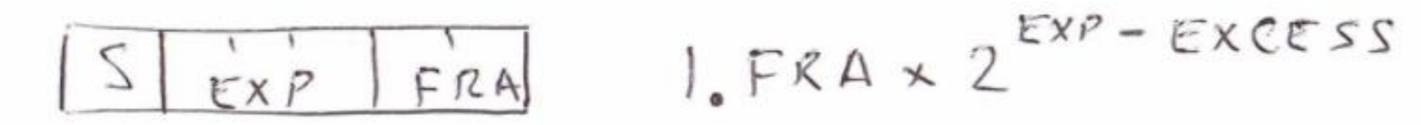
Scientific Notation and its representation in binary

- ***Avogadro's Number: 6.022 x 10^23***
- ***Where is the Decimal Point***
  - ***What determines where the decimal point is?***
  - ***Ergo, "floating point"***

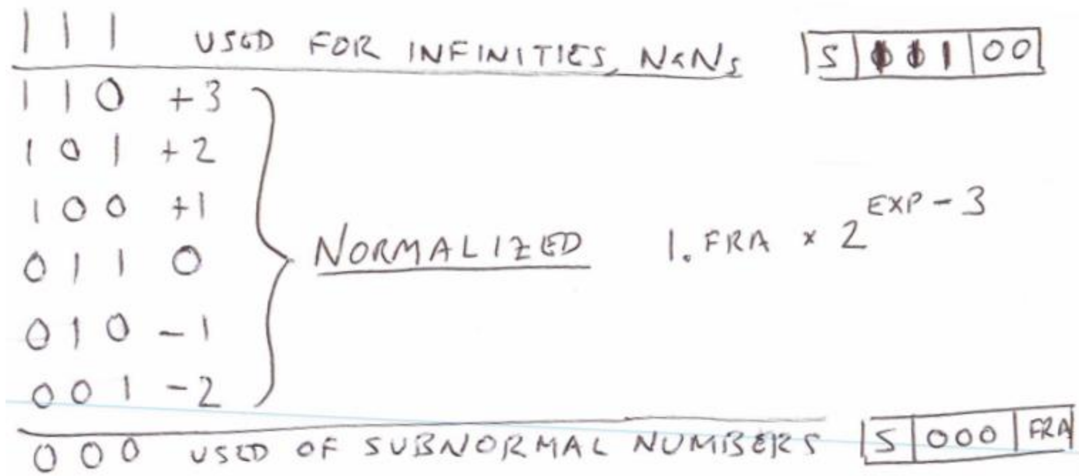
- ***Bits for range vs bits for precision***
  - ***Von Neumann's Quip***

- ***Binary Representations***
  - ***32 bits: 1 bit sign, 8 bits exponent, 23 bits of fraction***
  - ***64 bits (the default): 1 bit sign, 11 bits exponent, 52 bits fraction***
  - ***16 bits (recently for graphics): 1 bit sign, 5 bits exp, 10 bits fra***

My 6-bit floating point data type

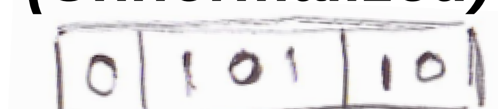


- ***Excess code for range (excess is 011, i.e. 3)***



- ***Signed-magnitude for precision***
  - ***Redundant most significant bit (because radix = 2)***

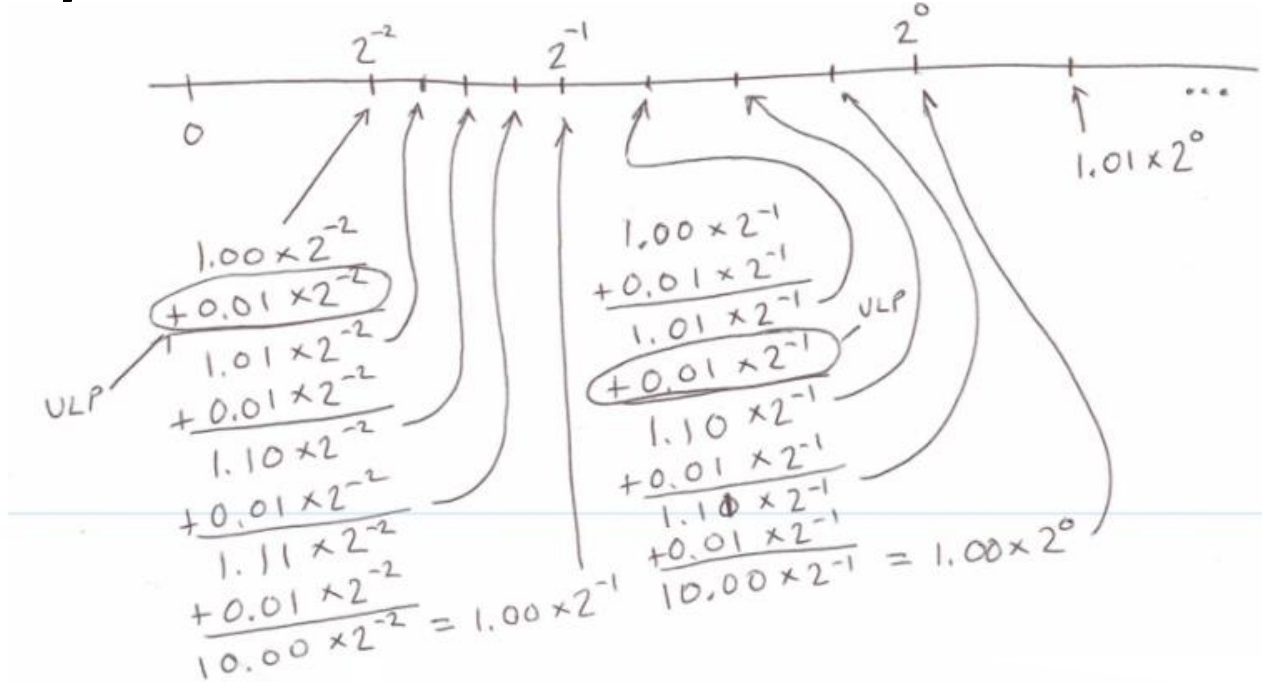
- ***An example: 6 5/8. 110.101 (Unnormalized)***
  - ***Normalize it: 1.10101 x 2^2***



  - ***Store in memory: Subsequent read from memory: 1.10 x 2^2 = 6***
    - ***We lost 5/8 because we had 2 fraction bits, but we needed 5 fraction bits***

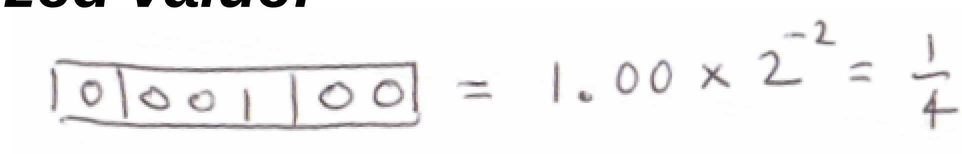
My 6-bit floating point data type

- ***Exact representations on the real line***



- ***Maximum value:***

- ***Minimum normalized value:***



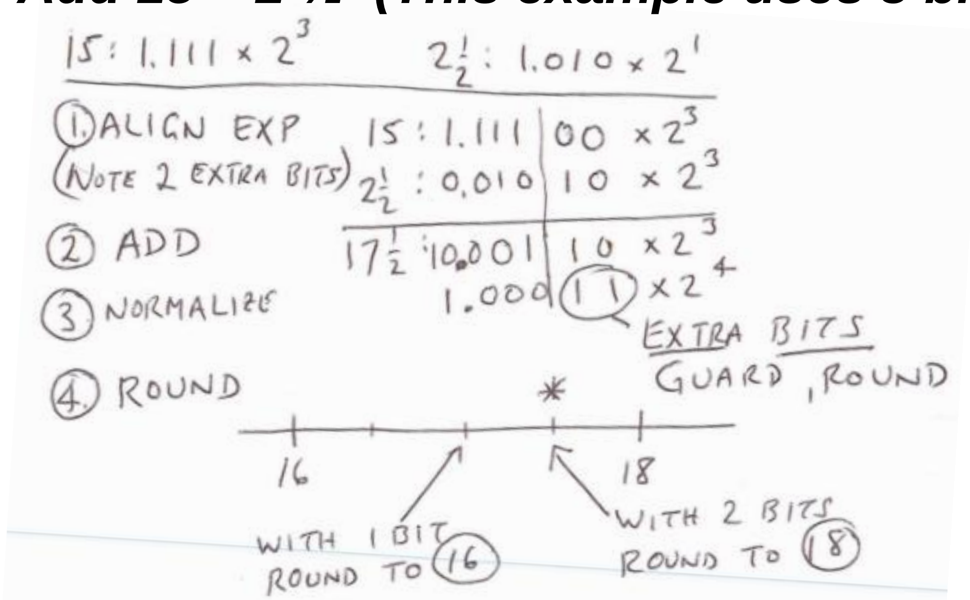
Rounding

- ***Why, When***
  - ***Why: when a value can not be represented exactly***
  - ***When: Often, most values can not be represented exactly***
  - ***Example (with our 6 bit floating point): 1.011 (1 3/8)***

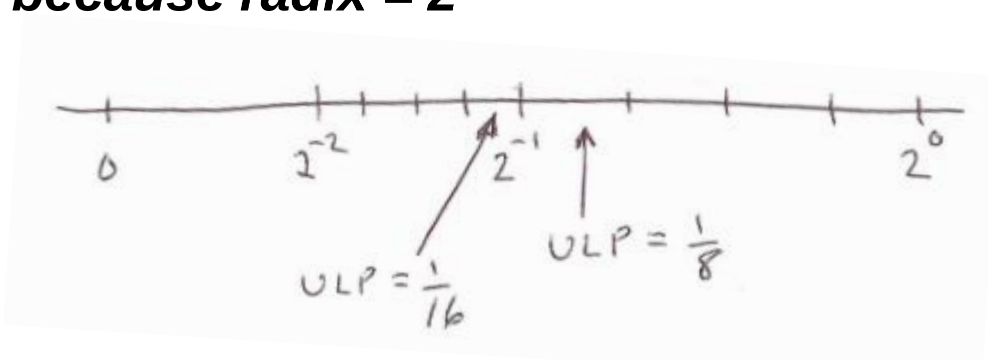
- ***Rounding Modes***
  - ***Round up: 1.10 (1 ½)***
  - ***Round down: 1.01 (1 ¼)***
  - ***Round to zero: 1.01 (1 ¼)***
  - ***Unbiased Nearest: 1.10 (1 ½)***
    - ***Why not 1.01 (1 ¼)?***
    - ***1 ¼ is just as "near" to 1 3/8 as 1 ½ is***
    - ***Unbiased → when equal, round to the value with 0 in the ULP***

Rounding

- ***Guard, Round, Sticky bits***
  - ***Extra bits carried along to help in rounding (1 bit or 2 bits?)***
  - ***Example: Add 15 + 2 ½ (This example uses 3 bits of fraction!)***



- ***Wobble (since 1 ULP < 2^k is half the size of 1 ULP > 2^k)***
  - ***Best case, because radix = 2***



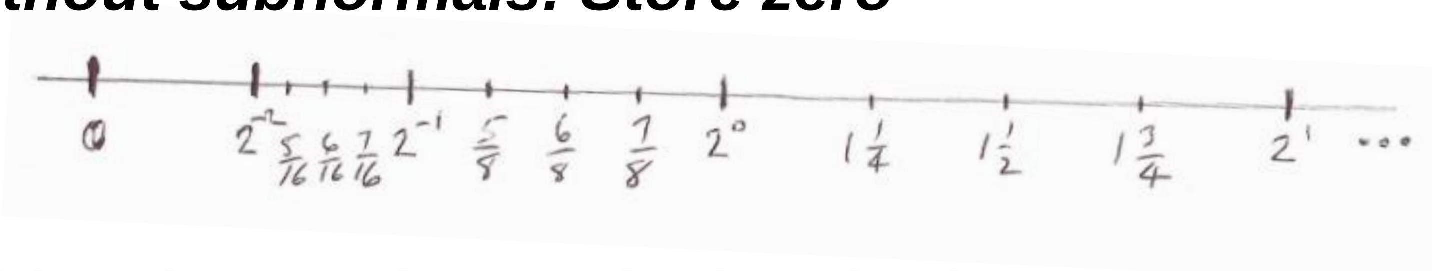
Infinity

- ***Exact vs Overflow***
  - ***(finite operands) → infinite results***
  - ***Examples: tan(90 degrees), 5 divided by 0***
- ***Infinity is NOT the same as undefined***
  - ***Example: continued fraction expansion***
  - ***Simple examples:***
    - ***infinity + 7 = infinity***
    - ***infinity + infinity = infinity***
    - ***5 divided by infinity = 0***
  - ***Code example***
    - ***X=5***
    - ***Y=0***
    - ***Z=X/Y***
    - ***W= arctan(Z)***

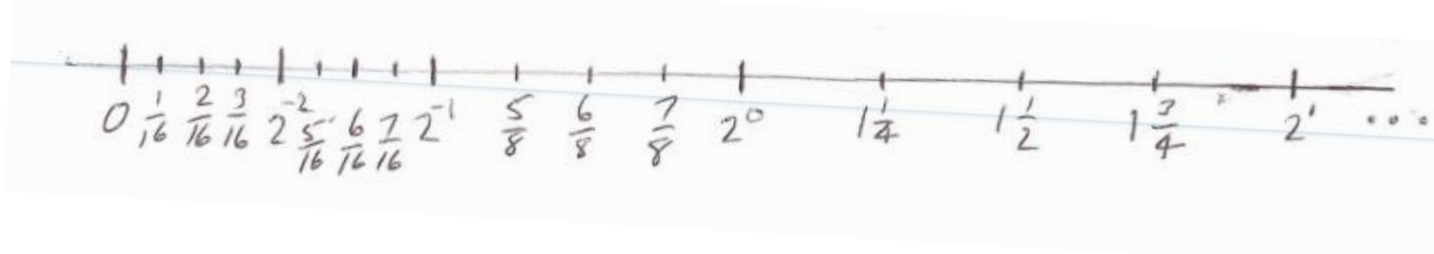
Subnormals

- ***Why?***
  - ***Underflow vs inexact discrepancy was unacceptable***
    - ***1 divided by underflow produces infinity***
- ***Tradeoff***
  - ***Subnormals provide gradual underflow***
    - ***Underflow is no worse than inexact***
  - ***Cost is loss of precision***

- ***Without subnormals: Store zero***



- ***With subnormals: Gradual underflow***



Not a Number (NaN)

- ***Examples***
  - ***Infinity minus infinity, infinity divided by infinity, 0 divided by 0***
  - ***arcsin(2), sqroot (negative number)***
  - ***A function that asymptotes***
- ***It was here before IEEE Floating Point***
  - ***Supercomputers had them, for example***
- ***The difference:***
  - ***IEEE Floating Pt allows exception handlers to be involved***
  - ***Allows correction of the problem and continue processing***

Five Floating Point Exceptions

- ***What are they?***
  - ***Overflow**: too large to represent in normalized form)*
  - ***Underflow**: too small to represent as a subnormal number*
  - ***Inexact**: not a value that can be represented exactly)*
  - ***Divide by zero**: function (finite arguments) → infinity*
  - ***Invalid**: creation of a NaN*

- ***Quiet vs Signaling***
  - ***Quiet: sets a sticky bit, handled under program control***
    - ***For example, usual way to deal with "inexact"***
  - ***Signaling: takes an exception to deal with the problem***
    - ***For example, usual way to deal with "NaNs"***

Arigato!