

Overview: This is a graduate level course on mathematical methods for network science. The area of network science concerns the theory, architecture and algorithm design for communications, control, and distributed applications over large networks (e.g., the Internet, wireless networks, social networks, economic networks). Applications that we will focus on in this course include network growth and evolution, strategic network formation, information and disease propagation through networks, consensus, learning and gossip, file distribution over P2P (peer-to-peer) networks (e.g., BitTorrent) and searching over social networks (e.g., Facebook). In this course we will focus on network structure and evolution, and dynamics over networks.

Pre-requisites: Probability and Stochastic Processes (EE 381J or equivalent).

Class Hours: Class will be held on **Tuesday and Thursday, 2:00 - 3:30 pm** in **ENS 126**. Office hours will be held after class in ENS 437 on Tuesday from 3:30 - 5 PM, and on Thursday from 11 - 12:30 PM.

Grading: Attendance is expected, and plus/minus grades will be assigned in this course. The breakdown is as follows:

- (i) Homeworks and paper summaries: 30%
- (ii) Midterm Exam: 20% (tentative: November 2, 2009)
- (iii) Final Project: 50%

References

- (i) Social and Economic Networks, M. Jackson, Princeton University Press, 2008.
- (ii) Random Graph Dynamics, R. Durrett, Cambridge University Press, 2007.
- (iii) Markov Chains: Gibbs Fields, Monte Carlo Simulation and Queues, P. Bremaud, Springer-Verlag, 1998.
- (iv) Networks out of control: Models and methods for random networks, M. Grossglauser and P. Thiran, Class notes available at <http://icawww1.epfl.ch/class-nooc/>
- (v) Connections: An Introduction to the Economics of Networks, S. Goyal, Princeton University Press, 2007.
- (vi) Complex Social Networks, F. Vega-Redondo, Cambridge University Press, 2007.

Course Policy: Course material will be available on Blackboard: <http://courses.utexas.edu>

You may discuss homeworks and papers with other students, but you are not allowed to copy from others. University disciplinary procedures will be invoked if any form of cheating is detected. Course and instructor evaluations will occur in the last week of class. Academic accommodations may be provided to students with disabilities. Please contact the Division of Diversity and Community Engagement, Services for Students with Disabilities (phone: 471-6259) for additional information.

Detailed Syllabus

1. **Section I - Introduction and Overview:** Course objectives, Basic graph theory notation, Review of stochastic processes (martingales, Markov chains)

Reading: Chapter 2, 3 - Jackson; Chapter 1 - Durrett

2. **Section II - Network Structure and Evolution:** In this section, we study network topologies, and the asymptotic properties that emerge based on the connectivity graph (2.a and 2.b). We then study network topologies that result from (i) imposing rules on how new nodes in the network “connect to” existing nodes (2.c), and (ii) network topologies that emerge if a utility function is imposed and nodes behave strategically (2.d).

- 2.a *Network Structure - Random Graphs:* $G(n, p)$ description, giant component, connectivity, diameter

Reading: Chapter 4 - Jackson; Section 2.1 - Durrett; Section 8.4 - Grossglauser and Thiran; Section 2.4 - Durrett

- 2.b *Network Structure - Small World Graphs:* Small world network, Strogatz and Watts model, analysis of small world networks

Reading: Sections 5.1, 5.2 - Durrett; (i) D. J. Watts and S. H. Strogatz, “Collective dynamics of small-world networks,” *Nature* 393:440-42(1998).

- 2.c *Graph Evolution:* Preferential attachment, emergence of power laws in networks, Polya’s Urn model, Degree distribution histogram using mean field calculations, analysis of degree distribution of a fixed node

Reading: Chapter 5 - Jackson; Sections 4.1, 4.3 - Durrett

- 2.d *Strategic Network Formation:* Pairwise stability, Pareto optimality, efficiency

Reading: Chapter 6 - Jackson, M. O. Jackson and A. Wolinsky, “A strategic model of social and economic networks,” *Journal of Economic Theory*, vol. 71, pp. 44 – 74, 1996.

3. **Section III - Dynamics over Networks:** In this section, we initially study tools (3.a) for understanding how messages, infection, and other types of “communication” occur over networks, and the effect of the network topology on the propagation of such messages (3.b). We then study properties of the network topology that enable messages and queries to navigate “quickly” through the network (3.c). Next, we study influence and consensus over networks (3.d) when nodes can communicate only with their neighbors (e.g., reaching a consensus opinion on the “likeability” of a politician over a social network; determining the average temperature over a sensor field). Finally, we study some recent papers which bring together the various tools and techniques (3.e).

- 3.a *Tools: Markov Chains and Random Walks on Graphs:* Markov chains and spectral graph theory, random walks on graphs

Reading: Markov chain notes (from Bremaud’s book); (i) L. Lovasz, “Random walks on graphs: A survey,” *Royal Society Mathematical Studies*, 2:1 – 46, 1993.

- 3.b *Diffusion through Networks:* Bass diffusion model, models for spread of infection and disease (SIR and SIS models), effects of network topology

Reading: Sections 7.1, 7.2 - Jackson; Section 3.5 - Durrett; (i) A. J. Ganesh, L. Massoulie and D. Towsley, The effect of network topology on the spread of epidemics, *Proc. IEEE Infocom*, 2005.

3.c *Searching and Navigation*: Searching and routing with local information, effects of network structure

Reading: Section 7.3 - Jackson; (i) J. Kleinberg, "Complex networks and decentralized search algorithms," In Proceedings of the International Congress of Mathematicians, 2006, (this is an overview paper); (ii) J. Kleinberg, "The small-world phenomenon: An algorithmic perspective," Proc. 32nd ACM Symposium on Theory of Computing, 2000; (iii) J. Kleinberg, "Small-World Phenomena and the Dynamics of Information," Advances in Neural Information Processing Systems (NIPS) 14, 2001.

3.d *Learning and Consensus*: DeGroot model and social influence models, consensus on networks

Reading: Chapter 8 - Jackson; Excerpts from Senata on non-homogeneous Markov chains; (i) J. N. Tsitsiklis, D. P. Bertsekas and M. Athans, "Distributed Asynchronous Deterministic and Stochastic Gradient Optimization Algorithms," IEEE Transactions on Automatic Control, Vol. 31, No. 9, 1986, pp. 803-812; (ii) A. Kashyap, T. Basar, R. Srikant, "Quantized consensus," Automatica, 43 (7), p.1192 - 1203, Jul 2007; (iii) B. Golub and M. O. Jackson, "How homophily affects learning and diffusion in networks", Stanford University technical report, January 2009.

3.e *Gossip and Distributed Algorithms*: Algorithms for file sharing, distributed scheduling, and broadcasting

Reading: (i) Boris Pittel, "On Spreading a Rumor," SIAM Journal on Applied Mathematics, Vol. 47, No. 1, Feb 1987; (ii) S. Boyd, A. Ghosh, B. Prabhakar and D. Shah, "Randomized gossip algorithms," IEEE Transactions on Information Theory, Volume 52, Issue 6, June 2006 Page(s):2508 - 2530; (iii) S. Deb, M. Medard and C. Choute, "Algebraic gossip: a network coding approach to optimal multiple rumor mongering," IEEE/ACM Transactions on Networking, Special issue on Networking and Information Theory, Volume 14, Pages: 2486 - 2507, June 2006; (iv) S. Sanghavi, B. Hajek, L. Massoulie, "Gossiping with Multiple Messages," IEEE Transactions on Information Theory Vol. 53, Issue 12, Dec 2007, pp. 4640-4654.

3.f *Resource Allocation and Games*: Price of anarchy and efficiency, utility maximization, price taking and price anticipation

Reading: Chapter 21, 22 - Nisan, Roughgarden, Tardos and Vazirani; (i) R. Johari and J.N. Tsitsiklis, "Efficiency loss in a network resource allocation game," Mathematics of Operations Research, 29(3):407 - 435, 2004.

4. **Section IV - Compressed Sensing and Network Applications**: (Guest Instructor: Constantine Carmanis) Brief review of convex optimization, convex approximations for low-rank and sparse matrices, applications in network inference
