Correction to "Unreliable Sensor Grids: Coverage, Connectivity and Diameter"

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1 Corrections

As pointed by S. Kumar et. al. in [1], the necessary conditions described in (2) and (3) in [2] are incorrect. A correct necessary condition was in fact given by us in (7) in the proof of Proposition 2.1 in [2]. In this note, we discuss the implications of this necessary condition on the rest of the results in [2]. First, we restate Proposition 2.1 of [2] with the appropriate corrections.

Proposition 2.1 Consider the random grid network with n nodes, with each node being active at time t with probability $p(n)$. Let $P_c(n)$ be the probability that the network covers the unit square. Then, we have

$$P_c(n) \quad \leq \quad \exp\left[-\frac{e^{-\theta(p(n))p(n)\pi r^2(n)n}}{4r^2(n)}\right], \tag{1}$$

where $\theta(p(n)) = -\log(1-p(n))/p(n)$. Further, a necessary condition for asymptotic coverage, i.e., $\lim_{n\to\infty} P_c(n) = 1$, is given by

$$c(n)\log(n)n^{c(n)\pi p(n)\theta(p(n))-1} \quad \stackrel{n\to\infty}{\to} \quad \infty, \tag{2}$$

where

$$c(n) \quad = \quad \frac{nr^2(n)}{\log(n)}. \tag{3}$$

$\diamond$

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From the above condition (2), we can obtain necessary conditions for $p(n)$ given $r(n)$ and vice-versa. However, the necessary condition (2) does not provide a simple relationship between $r(n)$ and $p(n)$ as the sufficient condition does in Proposition 2.2 of [2]. Therefore, next we compare the necessary and sufficient conditions for certain regimes of interest. Specifically, since we are primarily interested in sensors with a small range, we consider asymptotic regimes where the range is small and evaluate the reliability required to achieve coverage with high probability. As we will see from the examples below, the necessary and sufficient conditions provide bounds on $p(n)$ which are of the same order (as functions of n).

Example 1: From the topology of the grid, it is clear that we require $r(n) \geq \frac{1}{\sqrt{n}}$ for connectivity. Suppose that $r^2(n) = K/n$, for some $K \geq 1$. Then, $c(n) = K/\log(n)$, and the necessary condition on $p(n)$ becomes

$$Kn^{-\frac{K}{\log(n)}\pi\log(1-p(n))-1} \quad \rightarrow \quad \infty, \tag{4}$$

which implies that

$$\frac{-\pi K \log(1-p(n))}{\log(n)} \quad > \quad 1.$$

Thus, if $p(n) \rightarrow p < 1$ as $n \rightarrow \infty$, this condition cannot be satisfied. *Therefore, if* $r^2(n) \sim 1/n$, *then highly reliable nodes are required.* ∎

Example 2: Suppose that we have

$$r^2(n) \quad = \quad K\frac{\log(n)}{n},$$

which implies $c(n) = K$. Then, the necessary condition is given by

$$K\log(n)n^{K\pi p(n)\theta(p(n))-1} \quad = \quad f(n) \tag{5}$$

for some $f(n) \rightarrow \infty$. Further, suppose $p(n) \rightarrow p$. Then, the LHS (left-hand side) in (5) is

$$\sim K\log(n)n^{-K\pi\log(1-p)-1}$$

which goes to ∞ for sufficiently large p (we use $\sim$ to denote *asymptotic similarity* as $n \rightarrow \infty$). Thus, unreliable nodes may suffice. This is confirmed by the sufficient condition. From Proposition 2.2 of [2], the sufficient condition is given by

$$r^2(n)p(n) \quad > \quad \frac{4}{\pi}\frac{\log(n)}{n}.$$

For this example, this reduces to

$$Kp(n) \quad > \quad \frac{4}{\pi}$$

Thus, $p(n) > \frac{4}{K\pi}$ is sufficient, which is consistent with the necessary condition in an order sense. Thus, unreliable nodes (i.e., $p(n) < 1$) can provide coverage for $p(n)$ sufficiently bounded away from 0.

On the other-hand, if $p(n) \to 0$, we have for any fixed $\epsilon > 0$, there exists n_0 large enough such that for all $n \geq n_0$,

$$\log(1/(1-p(n))) \quad \leq \quad \epsilon$$

Hence, considering the necessary condition (4) again, we have in this case (for n large enough and by choosing ϵ to be small enough)

$$\begin{aligned} Kn^{-\frac{K}{\log(n)}\pi\log(1-p(n))-1} &= \frac{K\log(n)}{n}n^{K\pi\log(1/(1-p(n)))} \\ &\leq \frac{K\log(n)}{n}n^{K\pi\epsilon} \\ &\to 0 \end{aligned}$$

Thus, the necessary condition in (4) is not satisfied. Hence, we can conclude that *nodes with arbitrarily small reliability will not provide coverage if* $r^2(n) \sim \log(n)/n$. Both the necessary and sufficient conditions confirm this.

Example 3: We now consider the case where

$$r^2(n) \quad = \quad K\frac{(\log(n))^\alpha}{n},$$

for some $\alpha > 1$. Then, the LHS of the necessary condition (2) is given by

$$K\log(n)^\alpha n^{K\pi(\log(n))^{\alpha-1}p(n)-1}$$

in the regime where $p(n) \to 0$ (and hence, $\theta(p(n)) \to 1$). Thus, if

$$p(n) \quad = \quad \frac{1}{(\log(n))^{\alpha-1}},$$

and $K\pi > 1$, the necessary condition is satisfied. The sufficient condition (from Proposition 2.2 in [2]) requires that

$$Kp(n)\frac{(\log(n))^\alpha}{n} \quad > \quad \frac{4}{\pi}\frac{\log(n)}{n}$$

which implies that

$$p(n) \quad > \quad \frac{4}{K\pi}\frac{1}{(\log(n))^{\alpha-1}}$$

Thus, both the necessary and sufficient conditions require $p(n)$ to be of the order of $1/(\log(n))^{\alpha-1}$. In this case, *arbitrarily small reliability is sufficient.*

2 On Connectivity vs. Coverage

Since the conditions (2) and (3) in [2] are incorrect, the argument in Remark 3.1 is incorrect.

The sufficient condition (from (22) in [2]) for connectivity is given by

$$np(n)e^{-\frac{1}{2}\pi p(n) r^2(n) n} \quad \rightarrow \quad 0 \tag{6}$$

as $n \rightarrow \infty$.

Suppose that $p(n) = 1/n^2$ and $r^2(n) = K\log(n)/n$. Then, substituting in (6), we have

$$\begin{aligned}
\frac{n}{n^2}e^{-\pi\frac{1}{n^2}\frac{K\log(n)}{2n}n} &= \frac{1}{n}e^{-\frac{\pi K\log(n)}{2n^2}} \\
&= \frac{1}{n}\left(e^{\log(n)}\right)^{-\frac{\pi K}{2n^2}} \\
&= \frac{1}{n}n^{-\pi K/(2n^2)} \\
&\rightarrow 0,
\end{aligned}$$

as $n \rightarrow \infty$. Thus, the sufficient condition for connectivity is satisfied. While this choice of $p(n)$ and $r(n)$, may seem to imply that connectivity does not imply coverage, note that by allowing $p(n) = 1/n^2$, the expected number of active nodes in the network goes to zero, and thus the above example is a trivial case.

A study of connectivity vs. coverage is currently open for the non-trivial case where $np(n) \geq 1$, i.e., where the expected number of nodes in the network does not go to zero. Finally, we refer the reader to [3] which provides some tighter results.

References

[1] S. Kumar, T. H. Lai, J. Balogh. On k-Coverage in a mostly sleeping sensor network. *Proceedings of ACM Mobicom*, September, 2004.

[2] S. Shakkottai, R. Srikant and N. Shroff. Unreliable sensor grids: Coverage, connectivity and diameter. *Ad Hoc Networks*, 2005.

[3] V. Bhandari and N. Vaidya, Personal Communication.