MEASUREMENT OF THE DYNAMIC RESPONSE OF A CONTACT PROBE THERMOSENSOR IN CONDUCTIVE MEDIA

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ABSTRACT

This paper describes a method for characterizing the step response of a thermistor probe embedded in a low-conductivity solid. We define the “step response” as the dynamic response of a finite-size thermosensor instantaneously plunged into an infinite homogeneous conductive solid. The final goal of this research is to evaluate and enhance the time-dependent response of contact-type thermosensors. We will use the step response as the parameter for optimizing the probe time-dependent behavior. Although our research focuses on thermistors, the results could be applied to other contact-type sensors like thermocouples and RTD’s.

Currently, there is no direct way for determining the step response of the probe in such a case, since the probe can not be instantaneously plunged into a solid. In this paper, we describe an indirect experimental method for determining the step response of the probe. It is achieved by self-heating the thermistor and analyzing its temperature response. The success of this approach results from the fact that the heat transfer processes controlling self-heating are the same as the processes controlling the step response.

In this paper, we present an analytical expression for the step response of a spherical probe in a conductive solid. A relationship between the step response and the thermistor response to a step power self-heating is developed. Finally, a simple experimental method for determining the step response from the self-heating response is presented.

INTRODUCTION

The typical way to characterize the transient response of a temperature probe is the water-plunge test. In this test, the sensor, at a certain temperature, is plunged into water at another temperature flowing at a standard speed. Since this test involves the response of the sensor to a sudden change of the temperature surrounding the sensor, the plunge response is often called step-response.

The water-plunge test, however, gives limited information concerning the behavior of the probe in the actual measurement situation, since the test conditions are generally different from the measurement conditions. The speed and the thermal properties of the fluid surrounding the probe in the measurement may be different from the speed and thermal properties of the fluid in the test. Differences are more significant when the measurement is performed by a probe embedded in a solid. In this case, the probe response strongly depends on the thermal properties of the medium in which the probe is located (Valvano and Yuan, 1992). Thus, the water-plunge test gives a response which is significantly different from the response of a probe embedded in a solid. The intuitive way to get the true response of a probe in this case would be to "plunge" the thermistor into a solid with similar thermal properties. Agar-gelled water can be used to simulate tissue. However, this test is not practical in most cases.

A typical application of the conduction dominated environment is the thermal response of an oral thermometer. Other application include externally heated tissue using laser, ultrasound, or EM.

It is clear from the previous discussion that a method to determine the actual time response of a temperature probe in tissue would be useful. A great deal of research and development concerning methods for the *in situ* measurement of the time response of temperature sensors has been performed by Kerlin *et al.* (1980, 1981, 1984, 1982(a), 1982(b)). However, Kerlin's methods were specific for sensors in convective media.

NOMENCLATURE

a = thermistor radius (cm)
b = $\sqrt{\alpha_b/\alpha_m}\ k_m/k_b$
c = $1 - k_m / k_b$
c_b = specific heat of the probe material
k_b = thermal conductivity of probe bead (W/cm K)
k_m = thermal conductivity of medium (W/cm K)
Q = heat deposited in the probe (J/cm^3)
r = radius coordinate (cm)
t = time (s)
T = temperature rise inside the bead (^oC)
T_o = reference initial temperature of medium (K)
T_b = temperature rise of probe (^oC)
T_m = temperature rise of medium (^oC)
T_{plunge} = average temperature inside the bead (^oC)
T_{self} = average temperature inside the bead during self-heating
$T_{superp.}$ = temperature due to superposition of plunge responses
y = integration variable
α_b = thermal diffusivity of probe bead (cm^2/s)
α_m = thermal diffusivity of medium (cm^2/s)
Γ = rate of volumetric heat generation (W/cm^3)
ΔT_{imp} = increment in temperature due to unit impulse generation
λ = integration variable for convolution operation
ρ_b = density of the probe material

THE STEP RESPONSE OF A SPHERICAL TEMPERATURE PROBE EMBEDDED IN TISSUE

The probe is modeled by a sphere of radius *a*, thermal conductivity k_b and thermal diffusivity α_b. The medium is modeled as an infinite medium with thermal conductivity k_b and thermal diffusivity α_m. The temperature variables $T_b(r,t)$ and $T_m(r,t)$ are referred to an initial basal temperature in the probe (T_0):

$$T_b(r,t) = T_{b,0}(r,t) - T_0 \quad (1)$$

$$T_m(r,t) = T_{m,0}(r,t) - T_0 \quad (2)$$

The differential equation governing the system is the heat transfer equation in spherical coordinates applied to the thermistor and the medium:

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial}{\partial r} T_b(r,t)\right) = \frac{1}{\alpha_b}\frac{\partial T_b(r,t)}{\partial t} \quad \text{for } 0 < r < a \quad (3)$$

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial}{\partial r} T_m(r,t)\right) = \frac{1}{\alpha_m}\frac{\partial T_m(r,t)}{\partial t} \quad \text{for } a < r < \infty \quad (4)$$

The initial conditions are described by:

$$T_b(r,0) = V \quad \text{for } 0 < r < a \quad (5a)$$

$$T_m(r,0) = 0 \quad \text{for } a < r < \infty \quad (5b)$$

The boundary conditions are described by:

$$T_b(a,t) = T_m(a,t) \quad \text{for } t > 0 \quad (6.a)$$

$$T_m(\infty,t) = 0 \quad \text{for } t > 0 \quad (6.b)$$

$$k_b \left.\frac{\partial T_b(r,t)}{\partial r}\right|_{r=a} = k_m \left.\frac{\partial T_m(r,t)}{\partial r}\right|_{r=a} \quad (6.c)$$

$$T_b(r,t) \text{ is finite at } r=0 \quad (6.d)$$

This system of equations was solved by using the Laplace Transform. The inverse Laplace Transform was calculated by contour integration. The solution is:

$$T(r,t) = -\frac{2abV}{\pi r}\int_0^{\infty} e^{-\frac{\alpha_b}{a^2}y^2 t}\,\frac{[\sin y - y\cos y]\sin(\frac{r}{a}y)}{[c\sin y - y\cos y]^2 + b^2 y^2 \sin^2 y}\,dy \quad (7)$$

This analytical solution was verified by comparing temperature values calculated using finite difference approximation.

A numerical integration of Eq. *(7)* was performed using the parameters in table 1.

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$a=0.05$ cm
$k_b=0.001$ W / cm K
$k_m=0.005$ W / cm K
$\alpha_b=0.001$ cm^2/ s
$\alpha_m=0.002$ cm^2 / s
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Table 1. Parameters used in example in Fig. 1.

The integration was performed for values of r from 0.001 to 0.05 cm in steps of 0.001, and for values of time from 0 to 1.5 seconds. The results are shown in Fig. 1.

Figure 1. The temperature distribution in a spherical thermistor bead during a step change in the temperature of the surroundings every 0.05 sec.

Figure 1 shows that initially the heat transfer is more intense in the outer radius of the thermistor, and after some time the heat transfer increases also in the inner part of the thermistor body. This happens in this example because the thermal diffusivity of the medium is higher than the thermal diffusivity of the thermistor. It makes the heat flux higher at the thermistor border, especially in the beginning when the temperature gradients are high. After some time, the temperature gradient at the border becomes lower, and the heat flow becomes less intense.

The temperature measured by the thermistor will be assumed to be the spatial average of the temperature within the spherical bead, described by Eq. *(7)*:

$$T_{plunge}(t) = \frac{1}{\frac{4}{3}\pi a^3}\int_0^a 4\pi r^2 T_b(r,t)\,dr \qquad (8)$$

Substituting Eq. *(7)* into Eq. *(8)*, switching the order of integration, integrating, and reorganizing the terms:

$$T_{plunge}(t) = \frac{6bV}{\pi}\int_0^\infty \frac{e^{-\frac{\alpha_b}{a^2}y^2 t}}{y^2}\cdot\frac{[\sin y - y\cos y]^2}{[c\sin y - y\cos y]^2 + b^2y^2\sin^2 y}\,dy \qquad (9)$$

where $T_{plunge(t)}$ is define as the probe "step response" temperature.

The plunge response was numerically calculated by using the same parameters as in Fig. *(1)*, and the result is shown in Fig. *(2)*. A best fit exponential curve is also shown in the picture. These result indicates that the probe step response can not be accurately modeled as a first order system. It can be shown that the solution could be approximated by a summation of several exponential functions (Carslaw and Jaegger, 1959).

Figure 2 - The plunge response of the thermistor.

RESPONSE OF A PROBE EMBEDDED IN TISSUE TO A STEP POWER SELF-HEATING

The method for *in situ* measurement of the plunge response of a thermistor uses the response of the coupled thermistor/tissue system to a step power self-heating applied to the thermistor. The model situation is the same as in section *(3.1)*: the thermistor probe is modeled by a sphere of radius *a*, thermal conductivity k_b, and thermal diffusivity α_b. The medium is modeled as an infinite medium with thermal conductivity k_m and thermal diffusivity α_m. A constant power with density Γ is applied to the thermistor at time $t \geq 0$. It is assumed that the self-heating is uniformly distributed in the probe.

A previous work, by Goldenberg (1952), has shown that the solution for the region inside the thermistor is given by:

$$T_b(t,r) = \frac{a^2\Gamma}{k_b}\left[\frac{1}{3}\frac{k_b}{k_m} + \frac{1}{6}\left(1 - \frac{r^2}{a^2}\right)\right] - \frac{2a^3b\Gamma}{k_b\pi r}\int_0^\infty \frac{e^{-\frac{\alpha_b}{a^2}y^2 t}}{y^2}\,\frac{[\sin y - y\cos y]\sin(\frac{r}{a}y)}{[c\sin y - y\cos y]^2 + b^2y^2\sin^2 y}\,dy \qquad (10)$$

A simulation of the self heating process using the same parameters (k_b, α_b, k_m, α_m) as Fig. 1, was performed, for Γ=10 W/cm^3. The results for the first 2 seconds are shown in Fig. *3*.

Figure 3.Temperature distribution in a probe subjected to step self-heating every 0.05 sec.

Similar to Eq. (8), the temperature indicated by the probe is considered to be the volumetric average temperature in the sphere:

$$T_{self}(t) = \frac{1}{\frac{4}{3}\pi a^3}\int_0^a 4\pi r^2 T_b(r,t)\,dr \qquad (11)$$

or

$$T_{self}(t) = \frac{1}{\frac{4}{3}\pi a^3}\int_0^a 4\pi r^2\left\{\frac{a^2\Gamma}{k_b}\left[\frac{1}{3}\frac{k_b}{k_m} + \frac{1}{6}\left(1 - \frac{r^2}{a^2}\right)\right] - \frac{2a^3b\Gamma}{k_b\pi r}\int_0^\infty \frac{e^{-\frac{\alpha_b}{a^2}y^2 t}}{y^2}\,\frac{[\sin y - y\cos y]\sin(\frac{r}{a}y)}{[c\sin y - y\cos y]^2 + b^2y^2\sin^2 y}\,dy\right\}dr \qquad (12)$$

Performing the integration and rearranging the terms obtains:

$$T_{self}(t) = \frac{\Gamma a^2}{3k_b}\left(\frac{1}{5} + \frac{k_b}{k_m}\right) + \frac{6a^2b\Gamma}{k_b\pi}\int_0^\infty \frac{e^{-\frac{\alpha_b}{a^2}t\,y^2}}{y^4}\,\frac{[\sin y - y\cos y]^2}{[c\sin y - y\cos y]^2 + b^2y^2\sin^2 y}\,dy \qquad (13)$$

There are apparent similarities between Eqs. *(9)* and *(13)*. Equation *(9)* and the transient part of Eq. *(13)* have a similar form; the difference is that there is a term y^2 in Eq. *(9)*, instead of y^4. A simulation of the volumetric average temperature in the thermistor using the same parameters as in Fig. *(3)* is shown in Fig. *(4)*.

Figure 4. The analytical solution for the average temperature in the thermistor bead.

EXPERIMENTAL METHOD FOR THE DETERMINATION *IN SITU* OF A THERMISTOR PLUNGE RESPONSE USING A SELF-HEATING METHOD

The plunge response of the probe can be determined from the step power self-heating response. If successful, we could determine the step response of the probe *in situ*, without the disturbing problems of probe motion.

Even with the similarities between Eqs. *(9)* and *(13)*, they are fundamentally different. The response in Eq. *(13)* is slower than the response in Eq. *(9)*. The reason for the different behavior is simple: in the first case the heat is only leaving the bead, and in the second case there is heat being added to the bead, as heat exits the bead, making the second process slower.

The key for finding a relationship between the two equations is the superposition property of linear time-invariant systems. The method consists of subdividing the problem involving the self-heating process in an infinite number of infinitesimal solutions of the plunge response. Let us define a probe impulse response in the following sense: an impulsive amount of heat is uniformly deposited all over the spherical bead. The hypothetical spatial unit impulse would instantaneously deposit the heat density of *1 Joule/cm^3*. This amount of heat will cause the instantaneous temperature rise in the whole sphere of:

$$\Delta T_{imp} = \frac{1}{\rho_b c_b} \qquad (14)$$

After the impulsive energy is deposited in the bead, causing an instantaneous rise (ΔT_{imp}) in temperature, the heat starts to spread out to the medium. It is important to notice that, except for a multiplicative constant, this impulse response will have the same shape as the step response in Eqs. *(7)* and *(9)*. There is, therefore, a switch in nomenclature from this point. The step (or plunge) response determined before will become an impulse response in this new context.

The decomposition of the self-heating problem into a series of infinitesimal plunge response problems can be intuitively understood as follows. The constant power generation could be divided as a sequence of discrete little packets of heat, delivered uniformly to the bead in a sequential way, as illustrated in Fig. *(5)*. In the figure, the first differential heat package causes a uniform raise in temperature, which starts spreading out of the bead. After a differential moment, Δt, another heat packet is delivered, causing a uniform temperature rise, which adds to the present temperature distribution. And the process repeats indefinitely. It is important to remember that the real situation is the limit as the time intervals and the energy packets tend to zero.

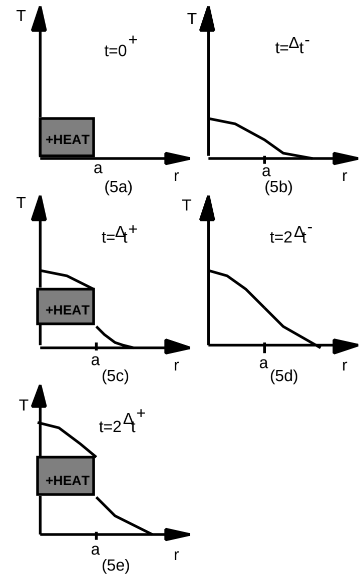

Figure 5.Illustration of the superposition theorem applied to the present problem. "+" and "-" mean time immediately after and before respectively.

With a little thought, one can show that the probe response is the convolution of the unit step response, with a function describing the generation of the instantaneous heat packets. Suppose that a uniform constant power is delivered at a rate Γ to the bead. We will divide the constant heat generation in a sequence of pulses of width Δt at intervals of Δt, as shown in Fig. 6.

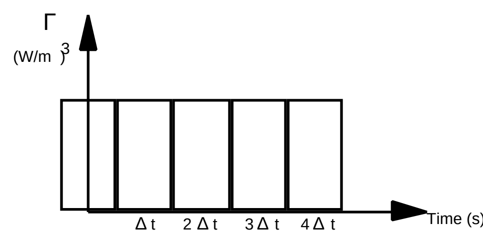

Figure 6.Decomposition of the linear temperature increase in infinitesimal blocks.

Each heat pack deposits the heat amount of $\Gamma\Delta t$. If Δt is small, the result of the heat deposition will be approximately the same as that of an impulse of intensity $\Gamma\Delta t$. Thus, the response to the heat packet at *t=0* would be a rise in temperature of :

$$\frac{\Gamma \Delta t}{\rho_b c_b} T_{plunge}(r,t) \qquad (15)$$

Similarly, the result for the heat packet at t=Δt will be the same, but delayed by Δt, that is:

$$\frac{\Gamma\Delta t}{\rho_b c_b} T_{plunge}(r, t-\Delta t) \tag{16}$$

The result for the heat packet at $n\Delta t$ will be:

$$\frac{\Gamma\Delta t}{\rho_b c_b} T_{plunge}(r, t-n\Delta t) \tag{17}$$

The net response will be the summation of all the individual responses, that is:

$$T_{superp.}(r,t) = \sum_{n=0}^{M} \frac{\Gamma}{\rho_b c_b} T_{plunge}(r, t-n\Delta t)\Delta t \tag{18}$$

where $T_{superp.}(r,t)$ is the result of the superposition of the heat packs (which will be shown to be equivalent to $T_{self}(r,t)$).

As $\Delta t \rightarrow 0$, the summation in Eq. (15) becomes the integral:

$$T_{superp.}(r,t) = \int_0^t \frac{T_{plunge}(r, t-\lambda)}{\rho_b c_b} d\lambda \tag{19}$$

which can be rewritten as:

$$T_{self}(r,t) = \int_0^t x(\lambda)\, h(r, t-\lambda)\, d\lambda \tag{20}$$

where $h(r,t)=T_{plunge}(r,t)/\rho_b c_b$ is the response to a unit power impulse, and $x(t)=\Gamma$ is the rate of heat generation in the bead.

Plugging Eq. (7), with $V = 1^oC$ into Eq. (19):

$$T_{superp.}(r,t) = \int_0^t \left[\frac{\Gamma}{\rho_b c}\left(\frac{2ab}{\pi r}\int_0^\infty e^{-\frac{\alpha_b}{a^2}y^2(t-\lambda)} f(r,y)\,dy\right)\right] d\lambda \tag{21}$$

where

$$f(r,y) = \frac{[\sin y - y\cos y]\sin(\frac{r}{a}y)}{[c\sin y - y\cos y]^2 + b^2y^2\sin^2 y} \tag{21b}$$

Reversing the order of integration, we get:

$$T_{superp.}(r,t) = -\frac{2abV}{\pi\rho c r}\int_0^\infty \left[\int e^{-\frac{\alpha_b}{a^2}y^2(t-\lambda)} f(r,y)\,d\lambda\right] dy \tag{22}$$

Solving the integral for λ, we get:

$$T_{superp.}(r,t) = \frac{2a^3bV}{\pi k_b r}\int_0^\infty \frac{f(r,y)}{y^2}dy - \frac{2a^3bV}{\pi k_b r}\int_0^\infty \frac{e^{-\frac{\alpha_b}{a^2}y^2 t} f(r,y)}{y^2}dy \tag{23}$$

Simplifying Eq. *(23)* results in Eq. *(13)*, which is the Eq. for the self-heated thermistor. Therefore, $T_{superb.}$ is the same as T_{self}. Thus, we have established the relationship between the plunge response and the self-heated response.

The next step is to use this result for designing a self-heating experiment for identification of the plunge response. To do so, note that Eq. (19) can be rewritten as:

$$T_{superp.}(r,t) = \Gamma u(t) * \left\{\frac{T_{plunge}(r,t)}{\rho_b c_b}\right\} \tag{24}$$

where the symbol "*" indicates convolution.

Taking the Laplace Transform of Eq. (24) gives:

$$\overline{T_{self}}(r,s) = \frac{\Gamma}{\rho_b c_b s}\overline{T_{step}}(r,s) \tag{25}$$

where the bar over T_{self} and T_{step} indicate Laplace Transform.

Isolating $\overline{T_{step}}(r,s)$, we obtain:

$$\overline{T_{step}}(r,s) = \frac{\rho_b c_b}{\Gamma} s\overline{T_{self}}(r,s) \tag{26}$$

Calculating the inverse Laplace Transform of Eq. (26) we obtain:

$$T_{step}(r,t) = \frac{\rho_b c_b}{\Gamma}\frac{dT_{self}(r,t)}{dt} \tag{27}$$

Thus, the plunge response is directly proportional to the derivative of the self-heated step response. Eq. (27) shows that the constant power self-heating was a good choice, since it leads to a very simple relationship.

Now, consider the expressions for the measured temperature in the plunge response and in the self-heating mode, Eqs. (9) and (13), in which the measured temperatures were assumed to be volumetric averages of the temperature distribution inside the bead. The volumetric integration performed during the evaluation of those equations does not affect the results in Eq. (27), and an expression relating the average temperatures $T_{plunge}(t)$ and $T_{self}(t)$ can be written as:

$$T_{plunge}(t) = \frac{\rho_b c_b}{\Gamma}\frac{dT_{self}(t)}{dt} \tag{28}$$

Thus, a simple experimental approach for identifying the plunge response of a probe is:

(i) Perform a step power self-heating experiment. This can be done by using by an smart electronic control that monitors the resistance and current passing through the thermistor, which supplies the necessary current to maintain a constant power generation.

(ii) Calculate the derivative of the signal using numerical methods. Use a method that eliminates high frequency noise.

(iii) The result is the plunge response multiplied by a constant dependent on

, and can be scaled.

EXPERIMENTS

A number of experiments have been performed in order to evaluate the validity of the method described. Some typical results are presented in this section. A Thermometrics P60 probe was self-heated with a constant power (10 mW), and the correspondent temperature was measured. The first measurement was performed in still water. The derivative was calculated and the result was normalized so that the response ranges from 0 to 1. A model, using equation *(9)*, for a spherical thermistor was run for different effective radiuses *a*. The assumed values of k_m and α_m for water were respectively $0.00613\ W/cm\ K$ and $0.0014678\ cm^2/s$, respectively. The result that best fits the experiment was chosen. The best effective radius was 0.031 inches. The self-heated measurement and the theoretical model with the best effective radius is shown in Fig. 7.

Figure 7. The model and the measured plunge response.

The measured plunge response had a fairly good agreement with the model. The possible causes of the errors will be discussed in the discussion section. A number of experiments were performed using glycerol and water, and using probes with different radii. The level of agreement was similar to that of Fig. 7.

DISCUSSION

Comparing the present model with a real thermistor system, we note that the model has several simplifying assumptions. Among the most important are:

(i) The measured resistance was assumed to be the volumetric average of the temperature distribution.

(ii) The thermistor leads deposit a uniformly distributed power Γ.

(iii) The thermal effect of the metallic leads was neglected.

(iv) The glass or epoxy coating that normally protects the thermistors was not considered.

(v) The thermal properties of the thermistor and the medium were assumed to be homogeneous.

The validity of Eq. (28) was demonstrated by using the known responses in Eqs. (9) and (13), for a simple spherical probe. However, the method is valid for a general geometry, provided that the assumptions above are valid, since the superposition theorem still remains valid for a general geometry.

We believe that the main limitation in this technique is due to the presence of the coating shell. The presence of the shell makes the true step response and the response found using the self-heating technique different. This is because in the self-heating situation the heat diffuses from the bead, crossing the shell, in an outward direction, and in the temperature measurement situation the heat starts diffuses from the tissue through the shell. This fact limits the practical applicability of the method to probes with a very protective coating. A model considering the coating shell is presently being developed in order to overcome this limitation.

CONCLUSION

This paper describes an *in situ* experimental method for determining the step response of a spherical temperature probe. A simple model describing the step response of a spherical probe embedded in a solid was presented. An experimental method for determining this step response by self-heating the thermistor was developed. Although the self-heating method was applied to a simplified spherical probe, it should still be valid for a general geometry, provided the assumptions in the model are met, because the superposition theorem is still valid.

A number of preliminary experiments were performed in order to assess the validity of the method. Although the results are encouraging, the method can undergo further development. The development of a more elaborated model would be required in order to account for the coating shell

REFERENCES

Carslaw, H. S., Jaegger, 1959, *Conduction of Heat in Solids*, 2n. ed, Clarendon Press, Oxford, ***-***.

Goldenberg, H, Tranter, M. A., 1952, "Heat flow in an infinite medium heated by a sphere", *British Journal of Applied Physics*, **3**:296-298.

Kerlin, T. W., 1980, "Temp Sensor", *Measurements & Control*, 122-127, April .

Kerlin, T. W., Hashemian, H. M., Petersen, K. M., 1981, "Time response of temperature sensors", *ISA Transactions*, **20(1)**:65-77.

Kerlin, T. W., Katz, E. M., 1990, "Temperature Measurement in the 1990s", Intech, august ***.***.

Kerlin, T. W., Miller, L. F., Hashemian, H. M., 1984, "In-situ Response Time Testing of Platinum Resistance Thermometers". *ISA Transactions*, **17(4)**: 71-88, .

Kerlin, T. W., Shepard, R. L., Hashemian, H. M., Petersen, K. M., 1982(a), "Response of installed temperature sensors", *Temperature Its Measurement and Control in Science and Industry*, **5**:1357-1366, .

Kerlin, T. W., Shepard, R. L., Hashemian, H. M., Petersen, K. M., 1982(b), "Response characteristics of temperature sensors installed in processes", *ACTA IMEKO*, Publishing House of the Hungarian Academy of Sciences, 95-103 .

Valvano, J. W., Yuan, D. Y., 1992, "Temperature errors when using probe-type transducers in laser irradiated biologic media", *Society for optical and Quantum Electronics, Proceedings of the International Conference on lasers* Houston, #0805, dec.