METHODOLOGY FOR MODELING THE RESPONSE OF TEMPERATURE PROBES IN CONVECTIVE MEDIA

Adson F. da Rocha
Department of Electrical Engineering
University of Brasilia
Brasilia, Brazil

Jonathan W. Valvano
Electrical and Computer Engineering
University of Texas at Austin
Austin, Texas

INTRODUCTION

This paper discusses the dynamic behavior of probes embedded in convective media during temperature measurements. It will be shown that in certain conditions the temperature measured by a probe can be written as the convolution of the true temperature with the impulse response of the probe. We present a general method to find the natural response of any kind of probe, and then we present results for a more realistic 1-D model for the thermistor probe in a thermodilution catheter. The results of these analyzes can be applied to enhance the dynamic response of temperature measurements made by probes in convective media.

BACKGROUND

Time-varying temperature measurements using probes is a very commonly performed task. It is important to note, however, that temperature measurements by a probe is an indirect process, and an error is always present. In a number of situations, the error involved in a measurement is so large, that the measurement is valid only as a qualitative observation. In general, a theoretical analysis of the system should be performed, in order to evaluate the magnitude of the error.

The study of transient errors is theoretically more difficult than steady state, because the transient heat equation is more complicated. However, the transient errors are much easier to assess experimentally. The relevant data is a function of time, and the data as a function of time can be easily and precisely obtained. To see this, notice that the independent variable (time) is very accurate, and that in an experiment the initial and final temperature can almost always be measured, and all the intermediate points as well. Because of the multitude of significant clinical situations which involve time-dependent thermal processes, this paper deals exclusively with the errors in transient state measurements.

A number of works discuss the dynamic behavior of temperature probes. Michalsky et al. [1991] presents a review on dynamic temperature measurements. They develop simplified first order models, and explore the step and sinusoidal responses of different systems. They also present more complex models using lumped elements to represent different components of the systems, in which each part of the probe was represented by an electrical analogue, composed of resistances and capacitances. A number of other works present models for the behavior of probes [Kerlin et al., 1982].

METHODOLGY

This section describes a methodology for developing a convolutional model for temperature probes in a medium with a time-independent convection coefficient, even though it can be non-uniform along the boundary. The requirement of a constant, non-uniform convection coefficient will make it possible to write a linear time-invariant relationship between the true and measured temperatures. The bulk temperature of the fluid will be the magnitude to be measured. The step response measured by the finite size probe can be written:

$$T_{meas}(t) = \sum_{n=1}^{\infty} d_n e^{-\lambda_n^2 t} \qquad (1)$$

where the coefficients d_n and λ_n are a function of the physical and thermal properties of the probe. Equation (1) illustrates the fact that, for a probe with a generic geometry, the solution can always be written as a summation of exponentials.

The next step is to use Duhamel's equation to write the convolutional response to a generic input $T_{in}(t)$.

$$T_{out}(t) = \int_{\tau=0}^{t} T_{in}(\tau) \frac{\partial T_{meas}(t-\tau)}{\partial t} d\tau \qquad (2)$$

In other words, $T_{in}(t)$ is the true fluid temperature and $T_{out}(t)$ is the probe response. The hard problem here is to solve the eigenvalue problem for a composite body, to find the coefficients of the exponentials. The only way to do this in complex geometries, is by using numerical methods.

MODEL FOR A THERMODILUTION CATHETER

The purpose of this section is to both illustrate the general method presented, and to provide a model for the temperature probe in a thermodilution catheter. The general method presented in the previous section can have analytical solutions for simpler cases, such as 1-D problems in composite bodies. In most of the cases, the probe behavior does not have a strictly 1-D characteristic. This section presents the model for the thermistor in a standard Swan-Ganz thermodilution catheter. A typical size is 8F meaning the catheter circumference is 8 mm. The thermistor probe is mounted near the surface, and it is protected by an epoxy shell.

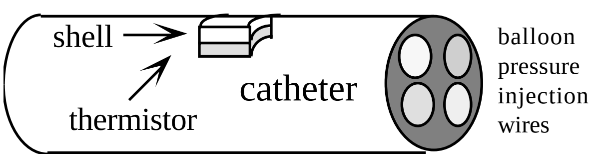

Figure 1 - Swan-Ganz thermodilution catheter

The thermistor probe is located a few centimeters from the catheter tip. Some simplifications are adopted. In the real catheter, the thermistor would not have the shape of a parallelepiped, but the shape of an oblate spheroid. Also the coating would not have uniform thickness. However, the model will provide a good qualitative understanding of the behavior of the probe. Another simplifying assumption will be that the catheter body is adiabatic. The derivative in Equation (2) is the impulse response of the probe. This response has an initial lag. Physically, the lag is caused by the presence of the protective shell. Mathematically, the lag is caused by the presence of one or more exponentials with negative coefficients, which cancel the positive exponentials for small values of time.

The model shows the catheter probe has a response that can be written as a summation of exponential responses. The most significant components are the first three components. The first harmonic is about 12 times the second, and 25 times the third. It is interesting to notice that the coefficients in Equation (2) define completely the response of the probe. If one is capable of measuring these values, then one will have the complete description of the behavior of the probe for any convection coefficient.

A good example of this kind of temperature measurement is the thermodilution curve. The catheter is inserted such that thermistor is located in the pulmonary artery, and a cold bolus of saline is injected in the right atrium. The resulting fluid temperature in the pulmonary artery, $T_{in}(t)$, is shown as the solid line in Figure 2. The dotted lines in Figure 2 are the thermistor responses approximated by Eq. (2). Figure 3 shows reasonable approximations can be obtained using only 3 components.

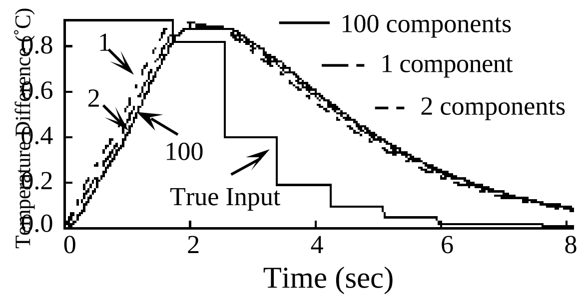

Figure 2- True input and responses.

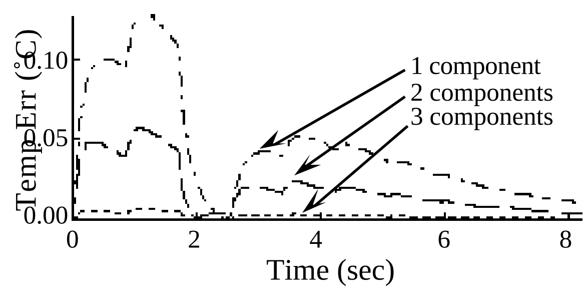

Figure 3- Temperature errors using 100 components as truth.

Armed with this model, we can now attempt the significant problem of signal enhancement. This model describes the smearing of the temperature signal because of the finite probe size. We believe it is possible to use this model to correct for this smearing, thereby recovering the true fluid temperature.

CONCLUSION

We presented an approach which defines the impulse response of a temperature transducer as a finite number of exponentials. The number of significant components depends on the signal. If the signal is slow enough, even a single-exponential approximation is good, since the others are integrated by the convolution process. The impulse response is a function of the transducer geometry and thermal properties. The output of the transducer can then be predicted for any convection coefficient by convolving the fluid temperature with this impulse response.

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